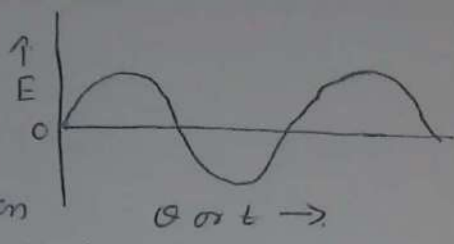
B.sc II nd Semester CBCS

Alternating Currents

Alternating current :-



A current which varies periodically, reversing in direction during every half a cycle is called an alternating current (AC)

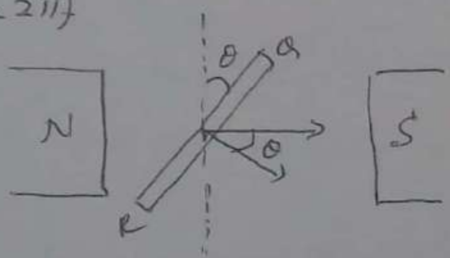
when a rectangular coil rotated in uniform magnetic field with uniform angular velocity, The coil intersects the magnetic ~~field~~ lines of force and an emf is induced in the coil (EM Induction)

During one complete rotation of the coil the induced emf reverses or alternates during every half cycle of revolution, During each of these half cycles the emf varies Sinusoidally as shown in fig. The current also varies sinusoidally the current is called alternating current.

Expression for Induced EMF :-

Consider a rectangular coil QR contains number of turns 'n' having area A rotates in a uniform magnetic field B with angular velocity $\omega = 2\pi f$

where $f \rightarrow$ frequency



Let $\theta \rightarrow$ angle of rotation in a small interval 't' sec Such that $\theta = \omega t$

The component of B $\perp^{r}$ to the coil in this position is $B.\cos\theta$

Flux linking the coil $= nAB\cos\theta$

$$\phi = nAB\cos\theta \qquad \text{—(1)}$$

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(nAB\cos\theta) \qquad (\theta = \omega t)$$

$$= nAB\omega \sin\omega t$$

$$E = E_0 \sin\omega t \qquad \text{—(2)}$$

where $E_0 = nAB\omega$ maximum or Peak value of emf

The Eqn (2) shows that the alternating emf is sinusoidal

If the Resistance of the coil is 'R'

Then current $I = \frac{E}{R} = \frac{E_0}{R} \sin\omega t$

$I = I_0 \sin\omega t$

Where $I_0 = \frac{E}{R}$ maximum value (Peak) of current

Average or Mean value of AC :-

It is defined as the average of the sum of the instantaneous value taken for half a cycle or half a period

i.e. mean value of emf $= \frac{2}{\pi} E_0$

IIIly mean value of current $= \frac{2}{\pi} I_0$

RMS value (Root mean square value) :-

It is defined as the square root of the mean (average) of the squares of the instanteneous values taken for a complete cycle or period.

rms value of emf $= \frac{E_0}{\sqrt{2}}$

rms value of current $= \frac{I_0}{\sqrt{2}}$

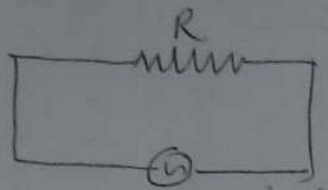
j - operator

The operator stands for $\sqrt{-1}$ and can be used to solve AC problems, $j^2 = -1$, $j^3 = -j$, $j^4 = 1$ . . .

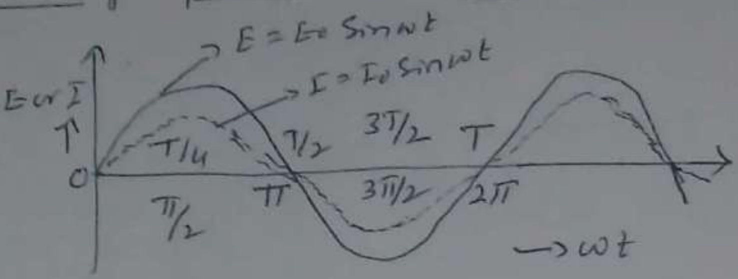
If a vector $\vec{a}$ is multiply by j^2, it becomes $-\vec{a}$ i.e. the vector is rotated by 180°, By multiplying $-\vec{a}$ by j^2 we get $\vec{a}$, which is further rotating 180° i.e. the operator j performed twice on a vector rotates it by 180°

If the operator j is performed once on a vector it is equivalent to a rotation of 90°, Similarly j^3 operator rotates the vector by 270°

Ac circuit containing pure Resistance (R) only



$E = E_0 e^{jwt}$



Consider a Circuit conteining a pure resistor of resistance R and an alternating emf $E_0 \sin \omega t$ applied across it, This voltage is the imaginary part of the complex number $E_0 e^{jwt}$ as shown in fig(1)

Thus in complex number representation

$$E = E_0 e^{jwt} \quad \text{——} (1)$$

If $I \rightarrow$ Instenteneous current in the Circuit at any time 't' then emf equation of the Circuit is

$$RI = E_0 e^{jwt} \quad \text{——} (2)$$

$$I = \frac{E_0 e^{jwt}}{R}$$

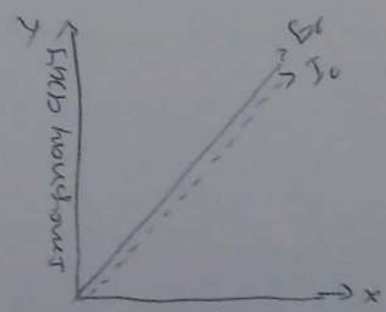
$$I = I_0 e^{jwt} \quad \text{——} (3)$$

where $I_0 = \frac{E_0}{R}$, the maximum value of current in the Circuit the Variation of current and emf through resistance 'R' shown in fig(2)

Comparing the two wave forms and corresponding Eqn (1) and Eqn (3) conclude that the current has the same wave form and frequency as that of the applied voltage Thus the current is in phase with applied emf as shown in fig(2) and fig(3) shows the complex number representation of voltage and current Through R. (Resistance)

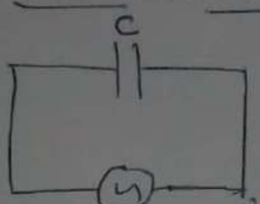
Here the impedence Z is real and equal to R so the admittence Y is given by

$$Y = \frac{1}{Z} = \frac{1}{R} \quad \text{——} (4)$$



Real axis
fig(3)

AC circuit containing pure capacitance (C) only



$E = E_0 e^{j\omega t}$

fig (1)

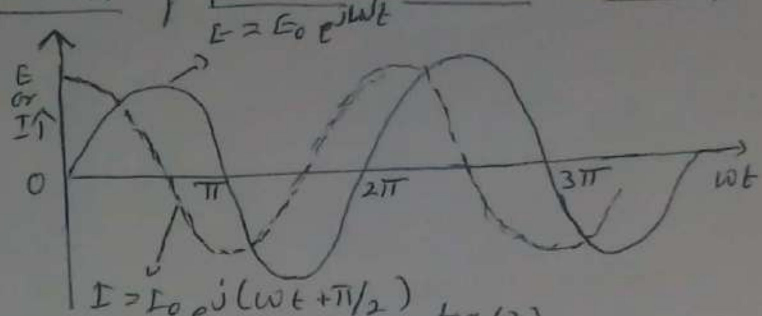


fig (2)

Consider a circuit containing pure capacitor of capacitance C with an alternating emf $E = E_0 \sin\omega t$ applied across it. as shown in fig (1)

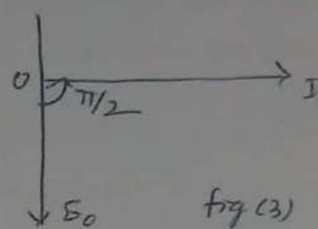


fig (3)

This voltage is the imaginary part of complex number $E_0 e^{j\omega t}$

$$E = E_0 e^{j\omega t} \quad \text{—— (1)}$$

The charge on capacitor at any time 't' is given by

$$q = C \times \text{applied emf} \qquad \left(\text{from } C = \frac{q}{V}\right)$$

$$= C\, e^{j\omega t}$$

The instantaneous current I due to the variation of voltage across the capacitor plates is given by

$$I = \frac{dq}{dt} = \frac{d}{dt}\left[C E_0 e^{j\omega t}\right]$$

$$= C j\omega E_0 e^{j\omega t}$$

$$= \frac{E_0 e^{j\omega t}}{\frac{1}{j\omega C}} = \frac{E}{\frac{1}{j\omega C}} \quad \text{—— (2)}$$

The quantity $\frac{1}{j\omega C}$ is called the complex capacitive reactance of the capacitor

$$I = \frac{E}{1/j\omega C} = \frac{E_0 e^{j\omega t}}{-j/\omega C}$$

$$= \frac{E_0 e^{j\omega t}}{\frac{1}{\omega C}\left[\cos\pi/2 - j\sin\pi/2\right]}$$

$$=$$

$$= \frac{E_0 e^{j\omega t}}{\frac{1}{\omega C} e^{-j\pi/2}} = \frac{E_0}{1/\omega C} e^{j(\omega t + \pi/2)}$$

$$= \frac{E_0}{X_C} e^{j(\omega t + \pi/2)} \qquad \text{——— (3)}$$

$$I = I_0 e^{j(\omega t + \pi/2)} \qquad \text{——— (4)}$$

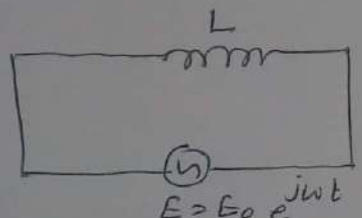
where $I_0 = \frac{E_0}{1/\omega C} = \frac{E_0}{X_C}$ is Peak value of current

Comparing equation (4) & (1) the current in the capacitance is ahead (leads) to the applied emf $\pi/2$ radian as shown in fig(3), The quantity $\frac{1}{\omega C}$ is called capacitive reactance of the capacitor of capacity C and denoted by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \qquad \text{——— (5)}$$

—x—

Ac Circuit containing pure Inductance (L) only



fig(1)

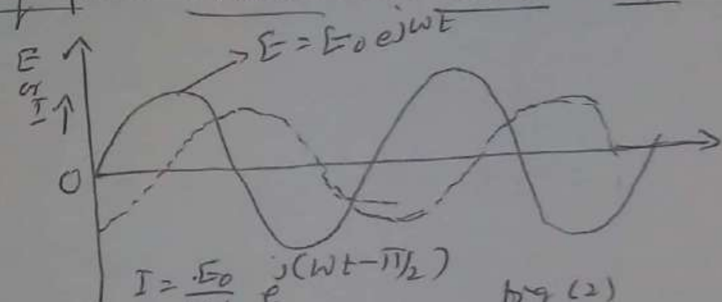


fig (2)

Consider a Circuit containing pure Inductance L with an alternating emf $E = E_0 \sin \omega t$ as shown in fig(1)

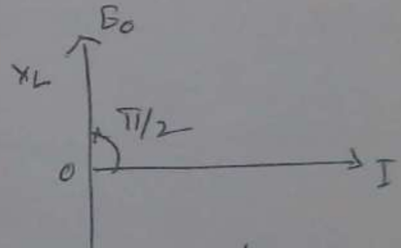


fig (3)

Thus the complex number representation we write as

$$E = E_0 e^{j\omega t} \qquad \text{——— (1)}$$

Let $I \rightarrow$ value of current at any time 't' and

$\frac{dI}{dt} \rightarrow$ rate of change of current at the same instant 't'

then emf induced in the inductor is given by $-L\frac{dI}{dt}$, the -ve sign indicates induced emf opposes the changes of current Then $E_0 e^{j\omega t} - L\frac{dI}{dt} = 0$ (by KVL)

$$dI = \frac{E_0}{L} e^{j\omega t} dt$$

Integrating the above eqn.

$$\int dI = \frac{E_0}{L} \int e^{j\omega t} dt + K.$$

where K is constant of integration and $K = 0$.

$$I = \frac{E_0}{L} \frac{e^{j\omega t}}{j\omega}$$

$$I = \frac{E_0}{j\omega L} e^{j\omega t} \text{ or } \frac{E}{j\omega L} \quad \text{——— (2)}$$

The quantity $j\omega L$ is the complex inductive reactance of the inductance L

Also $I = \frac{E}{j\omega L} = \frac{E_0 e^{j\omega t}}{\omega L[\cos \pi/2 + j \sin \pi/2]}$

$$= \frac{E_0 e^{j\omega t}}{\omega L \cdot e^{j\pi/2}}$$

$$I = \frac{E_0}{\omega L} e^{j(\omega t - \pi/2)} \quad \text{——— (3)}$$

$$I = I_0 e^{j(\omega t - \pi/2)} \quad \text{——— (4)}$$

where $I_0 = \frac{E_0}{\omega L}$ is the Peak value of current

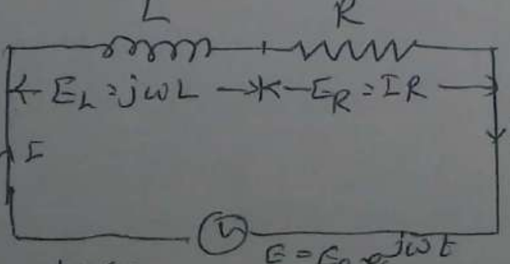
comparing eqn (4) and (1) the current lags the applied emf by phase angle $\pi/2$ radians in fig (3)

The quantity ωL is called inductive reactance of the inductance and is denoted by X_L

$$X_L = \omega L$$

$$X_L = 2\pi f L \quad \text{——— (5)}$$

Circuit with Inductance and Resistance in Series



fig(1)

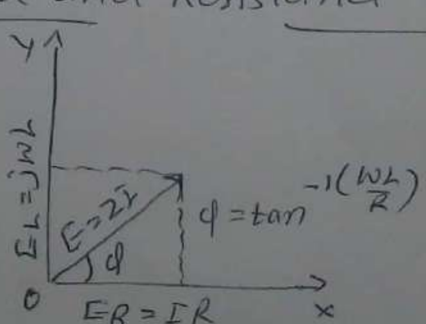
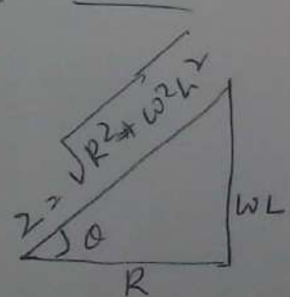


fig (2)



fig(3)

Consider an AC circuit containing a pure resistance R and pure Inductance L to which a complex emf $E = E_0 e^{j\omega t}$ is applied as shown in fig (1)

If $I \rightarrow$ current through the circuit
then potential drop across $R = E_R = IR$
the current in R is in phase with emf
Potential drop across $L = E_L = j\omega L I$
where $\omega L \rightarrow$ reactance due to Inductance X_L.
(Multiplication by) here E_L leads the current by a phase angle $\pi/2$. Hence that can represent $E_R = IR$ along the real axis and $E_L = j\omega L I$ along the imaginary axis of the complex plane as shown in fig(3)

$$E = E_R + E_L$$
$$= IR + j\omega L I$$
$$= I(R + j\omega L)$$
$$I = \frac{E}{R + j\omega L} = \frac{E_0 e^{j\omega t}}{R + j\omega L} \quad \text{——} (1)$$

put $R = Z\cos\phi$ and $L\omega = Z\sin\phi$ then

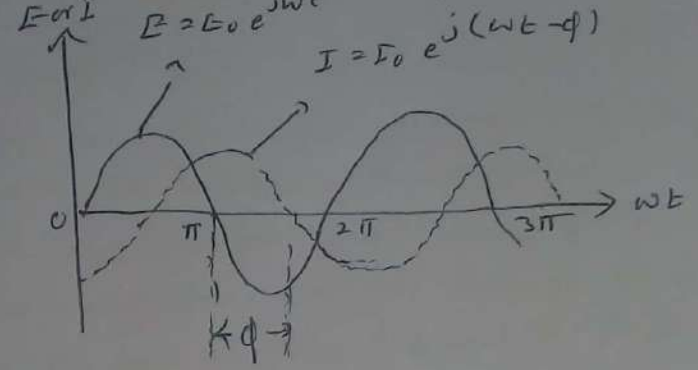
$$\tan\phi = \frac{\omega L}{R} \text{ and } Z = \sqrt{R^2 + \omega^2 L^2}$$
$$R + j\omega L = Z\cos\phi + jZ\sin\phi$$
$$R + j\omega L = Z e^{j\phi} \quad \text{put in (1)}$$
$$I = \frac{E_0 e^{j\omega t}}{Z e^{j\phi}} = \frac{E_0}{Z} e^{j(\omega t - \phi)} = I_0 e^{j(\omega t - \phi)}$$
$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + X_L^2}}$$

<u>phase Difference</u>:- when an alternating emf $E = E_0 e^{j\omega t}$ is applied to circuit containing R and L in series, the current through the circuit is
$$I = I_0 e^{j(\omega t - \phi)}$$

current lags w.r.t. to the emf, on other hand if the current flows through an LR Circuit the emf is given by

$$E = E_0 e^{j(\omega t + \phi)}$$

i.e. emf leads the current by an angle ϕ as shown in fig (2)



Impedence $Z = R + jL\omega$

$$= R + jX_L$$

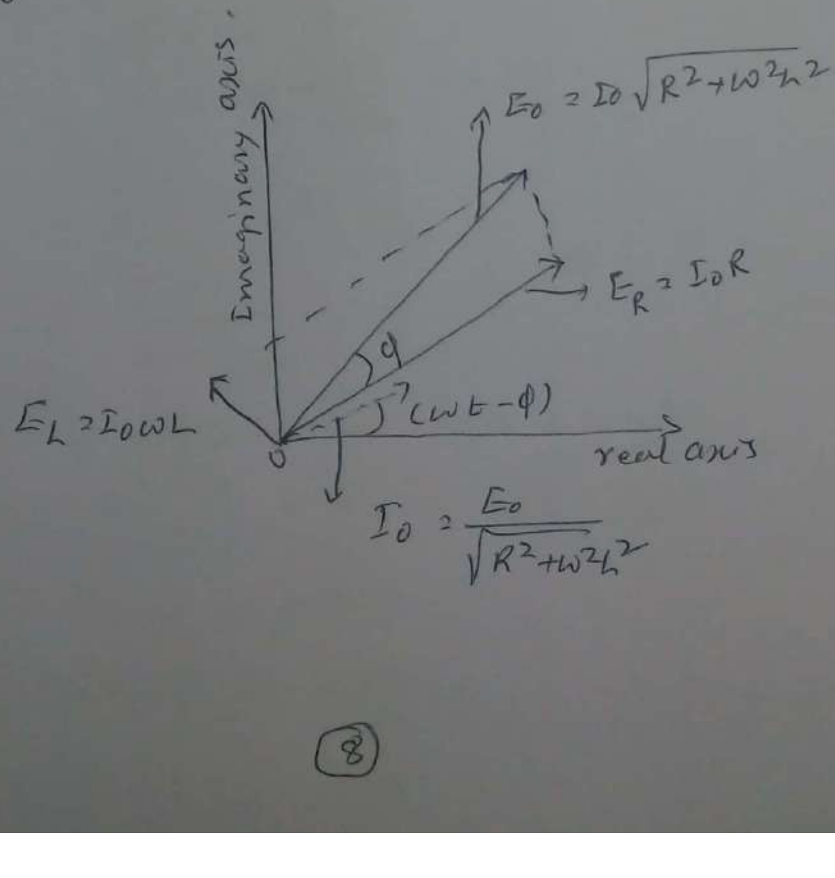
$\sqrt{R^2 + X_L^2}$ gives the modulus of Z

$$|\vec{Z}| = Z = \sqrt{R^2 + X_L^2}$$

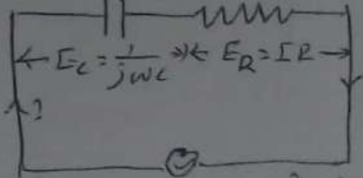
$$\phi = \tan^{-1}\frac{\omega L}{R}$$

$= \tan^{-1}\frac{X_L}{R}$ gives the argument of Z

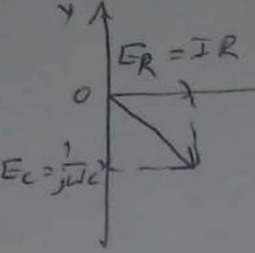
phasor diagram in complex representation as show below



Ac circuits with capacitance and Resistance in Series



fig(1) $E = E_0 e^{j\omega t}$



$\phi = \tan^{-1}\left(\frac{1/\omega C}{R}\right)$

fig(2)

Consider an ac circuit containing a pure resistance R and a pure capacitance C to which a complex emf $E = E_0 e^{j\omega t}$ is applied as shown in fig(1)

If $I \rightarrow$ the maximum current through the circuit then potential drop across $R = E_R = RI$. The current in R is phase with emf. The potential drop across $C = E_C = \frac{1}{j\omega C} I$.

where $\frac{1}{\omega C}$ is the reactance due to capacitance and is denoted by X_C

Multiplying $\frac{1}{j}$ implies that E_C lags with respect to the current by an phase angle $\pi/2$

Hence we can represent $E_R = IR$ along the real axis and $E_C = \frac{1}{j\omega C} I = -\frac{j}{\omega C} I$ along $-$ve direction of the imaginary axis of the complex plane as shown in fig(2)

$$E = E_R + E_C$$

$$= \cancel{IR + \frac{jI}{\omega C}} \; IR - \frac{jI}{\omega C}$$

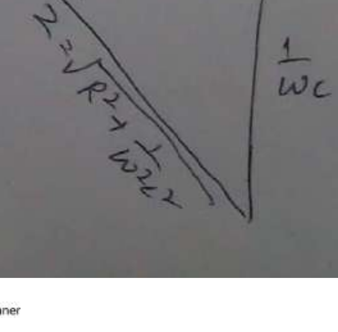
$$= I\left(R - \frac{j}{\omega C}\right)$$

$$I = \frac{E}{R - j/\omega C} = \frac{E_0 e^{j\omega t}}{R - j/\omega C} \qquad \text{——— (1)}$$

put $R = Z\cos\phi$ and $\frac{1}{\omega C} = Z\sin\phi$

$\tan\phi = \frac{1/\omega C}{R}$ and $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$\therefore R - \frac{j}{\omega C} = Z\cos\phi - jZ\sin\phi = Ze^{-j\phi}$



substituting in eqn (1)

$$I = \frac{E_0 e^{j\omega t}}{Z e^{-j\phi}} = \frac{E_0}{Z} e^{j(\omega t+\phi)} = I_0 e^{j(\omega t+\phi)}$$

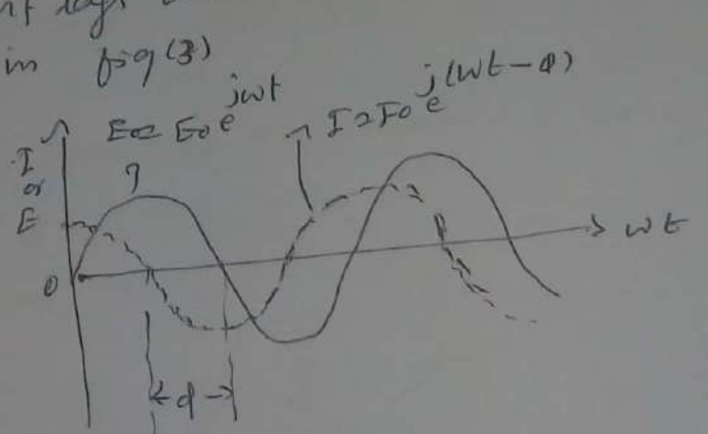
$$I_0 = \frac{E_0}{Z}$$

Phase Difference:-

When an alternating emf $E = E_0 e^{j\omega t}$ is applied to a circuit containing a resister and a capacitor in series, the current through the circuit is given by $I_0 = I_0 e^{j(\omega t+\phi)}$. The current therefore leads the emf. On the other hand if a current $I = I_0 e^{j\omega t}$ flows through the circuit the emf is given by

$$E = E_0 e^{j(\omega t-\phi)}$$

i.e. the emf lags with respect to the current by an angle ϕ as shown in fig (3)



Impedence !-

The complex Impedence of AC circuit

$$Z = R - \frac{j}{\omega C}$$

$$= R - jX_C$$

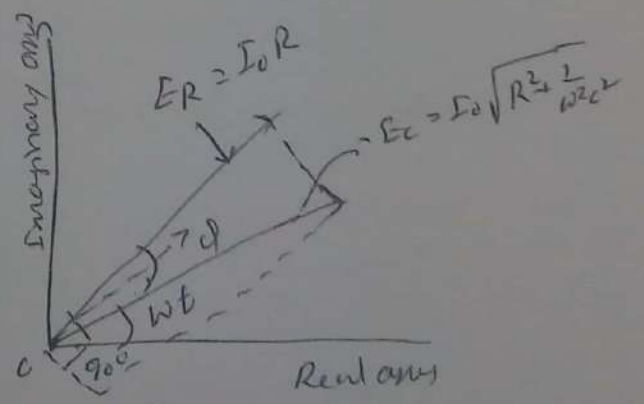
The modulus of $\vec{Z}$ will be

$$|Z| = Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$= \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\left[\frac{1/\omega C}{R}\right]$$

$$\phi = \tan^{-1}\left[\frac{X_C}{R}\right] \text{ gives the } Z$$



The above fig represent the phasor diagram in complex number representation

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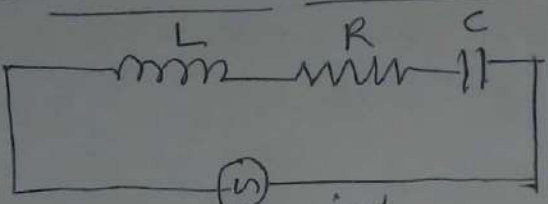
AC circuit with Inductance, capacitance and Resistance (LCR) in Series :-



$E = E_0 e^{j\omega t}$

Consider an ac circuit containing Inductance L connected in series with capacitance C and resistance R connected in series with Sinusoidal emf $E = E_0 e^{j\omega t}$ of frequency $f = \frac{\omega}{2\pi}$ as shown in fig.

Let $I \rightarrow$ instantaneous complex current. Then the voltage drop across R, L and C are, $E_R = IR$; $E_L = j\omega L I$, and $E_C = \frac{1}{j\omega C} I = -\frac{j}{\omega C} I$

In Vector form the emf equation of the circuit

$$E_R + E_L + E_C = E_0 e^{j\omega t}$$

$$I\left[R + j\left(\omega L - \frac{1}{\omega C}\right)\right] = E_0 e^{j\omega t}$$

$$I = \frac{E_0}{R + j\left(\omega L - \frac{1}{\omega C}\right)} e^{j\omega t} \quad \text{—— (1)}$$

$$I = \frac{E_0}{Z} e^{j\omega t} \quad \text{—— (2)}$$

where $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$ —— (3) is called as complex impedence of the circuit

Let $|Z| \cos\phi = Z \cos\phi = R$ —— (4)

and $|\bar{Z}| \sin\phi = Z \sin\phi = \left(\omega L - \frac{1}{\omega C}\right)$ —— (5)

Squaring and adding eqn (4) & (5) we get-

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad \text{—— (6)}$$

This gives the magnitude of the complex impedence

Now from eqn (4) and (5)

$$\cos\phi = \frac{R}{Z} \text{ and } \sin\phi = \left[\frac{\omega L - \frac{1}{\omega C}}{Z}\right], \tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} \quad \text{—— (8)}$$

Now from eqn (3), (4) and (5)

$$\vec{Z} = Z(\cos\phi + j\sin\phi) = Z e^{j\phi} \quad -(8)$$

From eqn (3) is given by $I = \frac{E_0}{Z} e^{j\omega t} = \frac{E_0}{Z} \frac{e^{j\omega t}}{e^{j\phi}} = \frac{E_0}{Z} e^{j(\omega t - \phi)} \quad -(9)$

$$I = I_0 e^{j(\omega t - \phi)} \quad -(10)$$

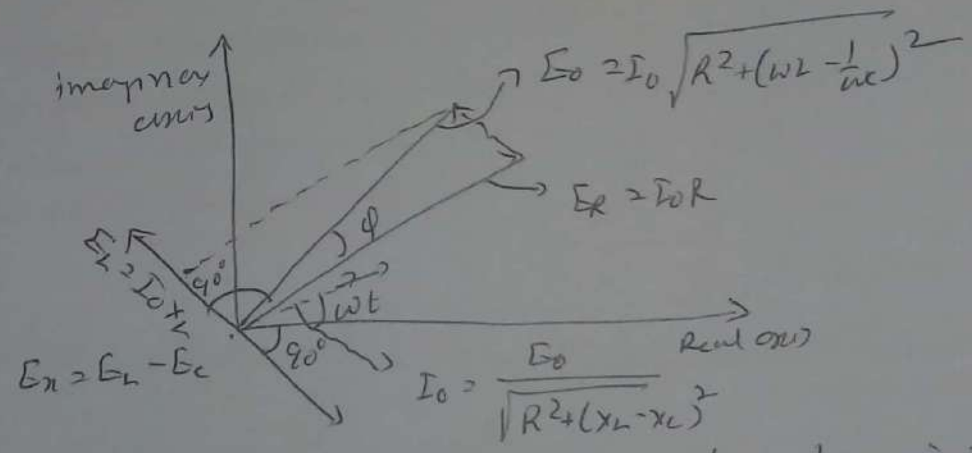
$$Z = \frac{E_0}{I_0} = \frac{E_0/\sqrt{2}}{I_0/\sqrt{2}} = \frac{E_{rms}}{I_{rms}} \quad -(12)$$

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \quad -(11)$$

The impedence of LCR Circuit can be defined as the ration of the rms value of emf to the rms value of current through it.

The equations (10) gives current lags the applied voltage in phase by an angle given by

$$\phi = \tan^{-1}\left[\frac{\omega L - \frac{1}{\omega C}}{R}\right]$$



This phase difference as to shown in above fig. in the complex plane, The voltage across the resistance E_R is in phase with the current while $E_X = E_L - E_C$ which is the net voltage across the combination of L and C leads the current by an angle $\pi/2$, The total voltage E_0 is shown leading the current by an angle ϕ

<u>Resonance</u>: case I:- when $\omega L > \frac{1}{\omega C}$ i.e. when the inductive reactance is greater then the capacitive reactance, ϕ is +ve or the current lags behind the voltage

<u>case II</u>:- $\omega L < \frac{1}{\omega C}$, ϕ is −ve, i.e. ~~the current and voltage are in phase with each other~~ The current leads the voltage

Case III :- $\omega L = \frac{1}{\omega C}$, $\phi = 0$, i.e. the current and voltage are in phase with each other, Then the LCR series circuit is said to be under resonance

Definition of Resonance :- It is the phenomenon in which inductive reactance is equal to capacitive reactance

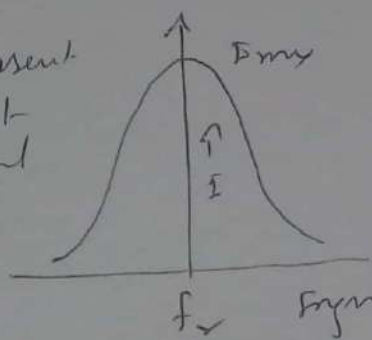
i.e. $\omega L = \frac{1}{\omega C}$; $\omega^2 = \frac{1}{LC}$, $\omega = \frac{1}{\sqrt{LC}}$

$$2\pi f = \frac{1}{\sqrt{LC}} \qquad (\omega = 2\pi f)$$

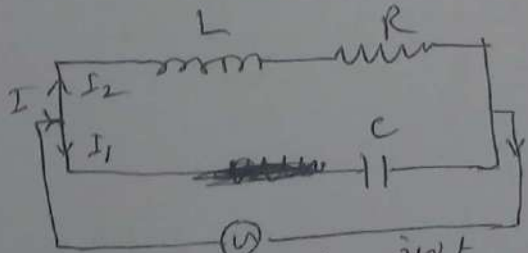
$$f = \frac{1}{2\pi\sqrt{LC}}$$

The frequency at which this occurs is called Resonant frequency

fig. shows the variation of current with frequency and the current is maximum at frequency equal to resonant frequency (f_r)



parallel Resonant Circuit (LCR parallel circuit)



Let an alternating voltage $E = E_0 e^{j\omega t}$ applied across the coil of inductance L of some negligible resistance R in parallel with capacitor C as shown in fig(1)

In this case $\quad I = I_1 + I_2$

If Z is the impedence of the parallel circuit

Z_1 and $Z_2 \rightarrow$ impedence of C and L-R branches.

Then $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$ —— (1)

$Y = Y_1 + Y_2$ where $Y = \frac{1}{Z}$

$I_2 = \frac{E}{Z_2}$, $I_1 = \frac{E}{Z_1}$

$\therefore I = I_1 + I_2$

From fig(1) $Z_1 = \frac{1}{j\omega C}$

$Z_2 = R + j\omega L$

$\frac{1}{Z_1} = j\omega C$ and $\frac{1}{Z_2} = \frac{1}{R + j\omega L}$ —— (2)

multiplying by $R - j\omega L$ we get -

$\frac{1}{Z_2} = \frac{R - j\omega L}{(R + j\omega L)(R - j\omega L)} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$ $(\because j^2 = -1)$

$\therefore$ Total impedence $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$ or $Y = Y_1 + Y_2$

$Y = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$

$= j\omega C + \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2}$

$Y = \frac{R}{R^2 + \omega^2 L^2} + \frac{j\omega C(R^2 + \omega^2 L^2) - j\omega L}{R^2 + \omega^2 L^2}$

$= \frac{R}{R^2 + \omega^2 L^2} + \frac{j\omega(CR^2 + C\omega^2 L^2 - L)}{R^2 + \omega^2 L^2}$ —— (3)

This can be written as

$Y = A + iB$

where $A = \frac{R}{R^2 + \omega^2 L^2}$ and $B = \frac{\omega(CR^2 + C\omega^2 L^2 - L)}{R^2 + \omega^2 L^2}$ —— (4)

Now $\tan\theta = \frac{B}{A}$ At resonance $\theta = 0$.

$B = 0 \therefore A \neq 0$. —— (5)

$\therefore CR^2 + C\omega^2 L^2 - L = 0$.

$C\omega^2 L^2 = L - CR^2 \rightarrow \omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$ $\therefore \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ —— (6)

$$2\pi f = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad \text{—} (7)$$

Q - Factor (Quality of a coil)

The quality or Q-value of a coil in a circuit is defined as the ratio of the Inductive reactance to ohmic resistance and is represented by the symbol 'Q'

If L and R are the inductance and resistance in the circuit to which an AC of frequency 'f' is applied.

the inductive reactance $= L\omega$

the ohmic resistance $= R$

Hence Q - factor of the coil $Q = \frac{L\omega}{R}$

At resonance, the voltage across the inductance is $I_0 \omega L$, where I_0 is current at resonance

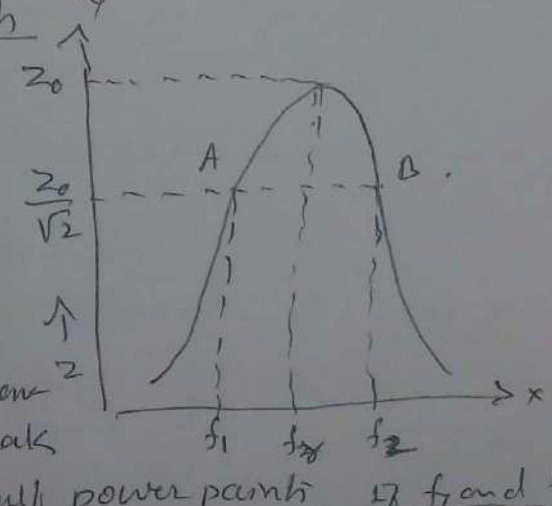
The voltage across resistance $= I_0 R$

Dividing, $\frac{\text{P.d across } L}{\text{p.d. across } R} = \frac{I_0 \omega L}{I_0 R} = \frac{\omega L}{R} = Q.$

Hence Q can also be defined as the ratio of pd across the inductance at resonance to the pd across Resistance

Band width

The band width of a resonator circuit is defined as the band of frequencies which between two point A and B (as shown in fig) on either side of the maximum frequency response curve when the impedance drops to $\frac{1}{\sqrt{2}}$ or 0.707 of its peak value. A and B are called half power points. If f_1 and f_2 are the frequencies, corresponding to A and B, Then $\boxed{BW = f_2 - f_1}$



Sharpness:- The sharpness of resonance is the ratio of BW of the circuit to its resonance frequency (f_r)

Sharpness of resonance $= \frac{BW}{f_r} = \frac{f_2 - f_1}{f_r}$

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