ELECTROMAGNETIC INDUCTION AND FARADAY'S LAW

Magnetic Flux

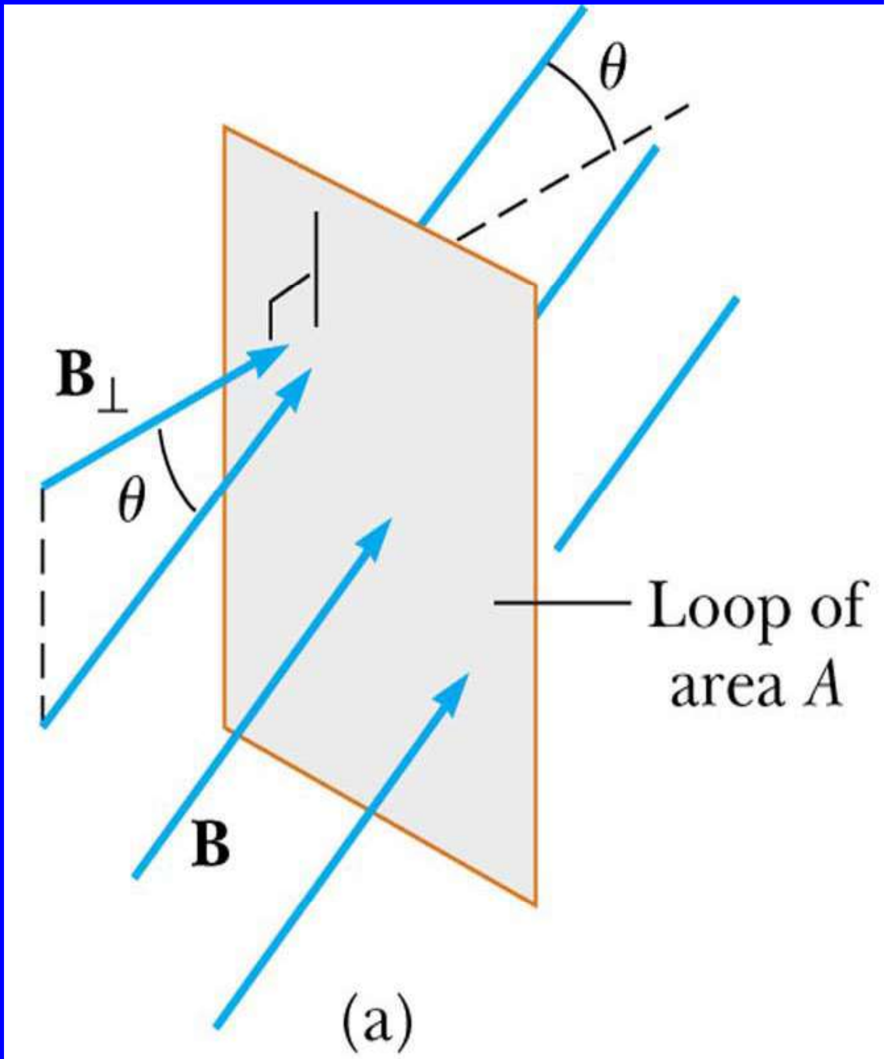
- The emf is actually induced by a change in the quantity called the *magnetic flux* rather than simply by a change in the magnetic field
- Magnetic flux is defined in a manner similar to that of electrical flux
- Magnetic flux is proportional to both the strength of the magnetic field passing through the plane of a loop of wire and the area of the loop

Magnetic Flux is proportional to both the strength of the magnetic field passing through the plane of a loop of wire and the area of the loop

- The loop of wire is in a uniform magnetic field B
- The loop has an area A
- The flux is defined as

$$\Phi_B = B_\perp A = B\,A\cos\theta$$

  - θ is the angle between B and the normal to the plane.
  - SI units of flux are $T\,m^2$ = Wb (Weber)



The induced emf in a wire loop is proportional to the rate of change of magnetic flux through the loop.

Magnetic flux:

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}.$$

Unit of magnetic flux: weber, Wb:

$$1\ \mathrm{Wb} = 1\ \mathrm{T \cdot m^2}.$$

Faraday's Law and Electromagnetic Induction

- The instantaneous emf induced in a circuit equals the time rate of change of magnetic flux through the circuit
- If a circuit contains one tightly wound loop and the flux changes by $\Delta\Phi_B$ during a time interval Δt, the average emf induced is given by *Faraday's Law:*

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt}.$$

Faraday's Law and Electromagnetic Induction

- If a circuit contains N tightly wound loops and the flux changes by ΔΦ during a time interval Δt, the average emf induced is given by *Faraday's Law:*

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

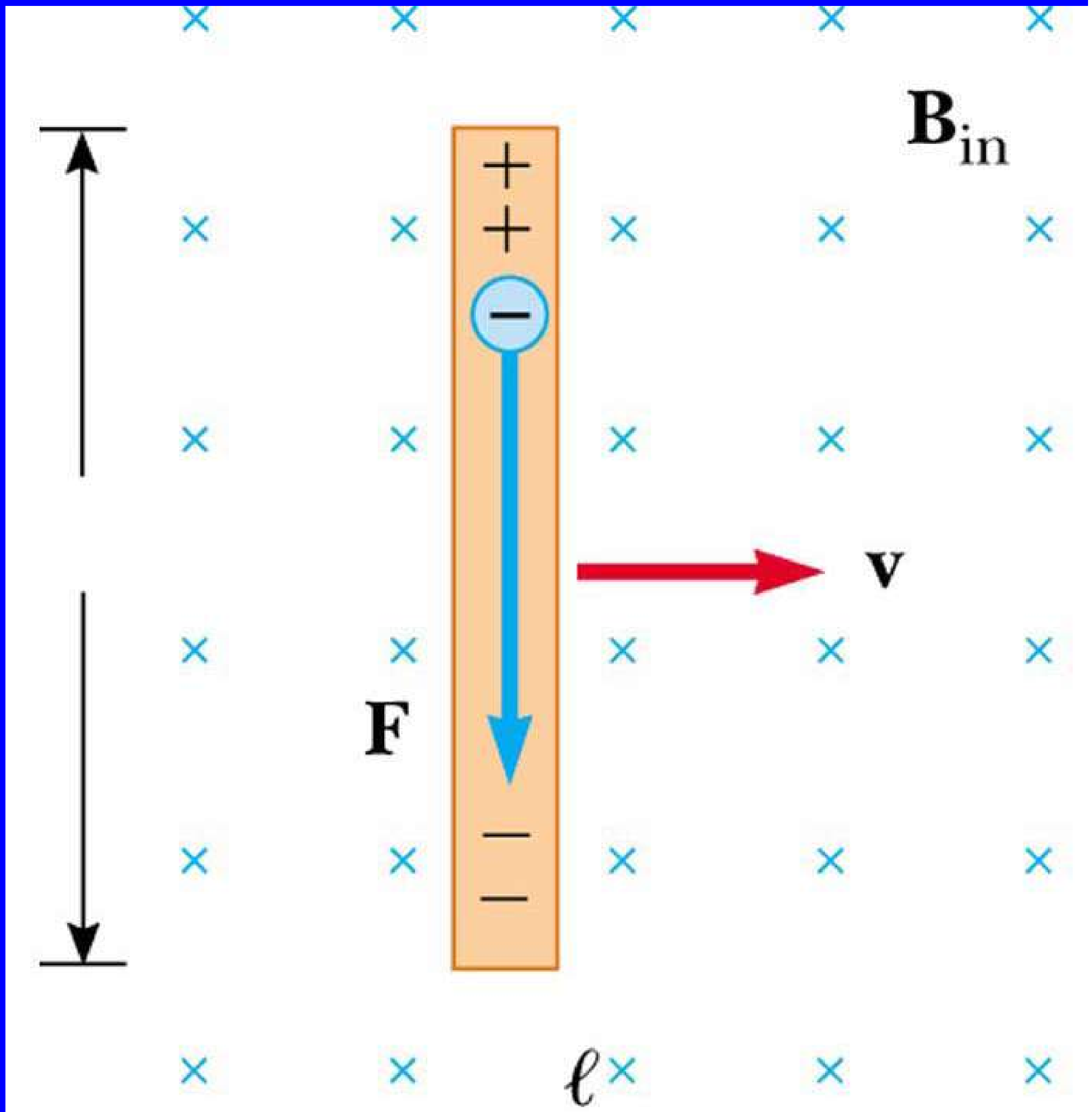
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}. \qquad [N \text{ loops}]$$

Faraday's Law and Lenz' Law

- The change in the flux, $\Delta\Phi$, can be produced by a change in B, A or θ
  - Since $\Phi_B = B\,A\cos\theta$
- The negative sign in Faraday's Law is included to indicate the polarity of the induced emf, which is found by *Lenz' Law*
  - The polarity of the induced emf is such that it produces a current whose magnetic field opposes the change in magnetic flux through the loop
  - That is, the induced current tends to maintain the original flux through the circuit

Application of Faraday's Law – Motional emf

- A straight conductor of length ℓ moves perpendicularly with constant velocity through a uniform field
- The electrons in the conductor experience a magnetic force
  - $F = q\,v\,B$
- The electrons tend to move to the lower end of the conductor



Motional emf

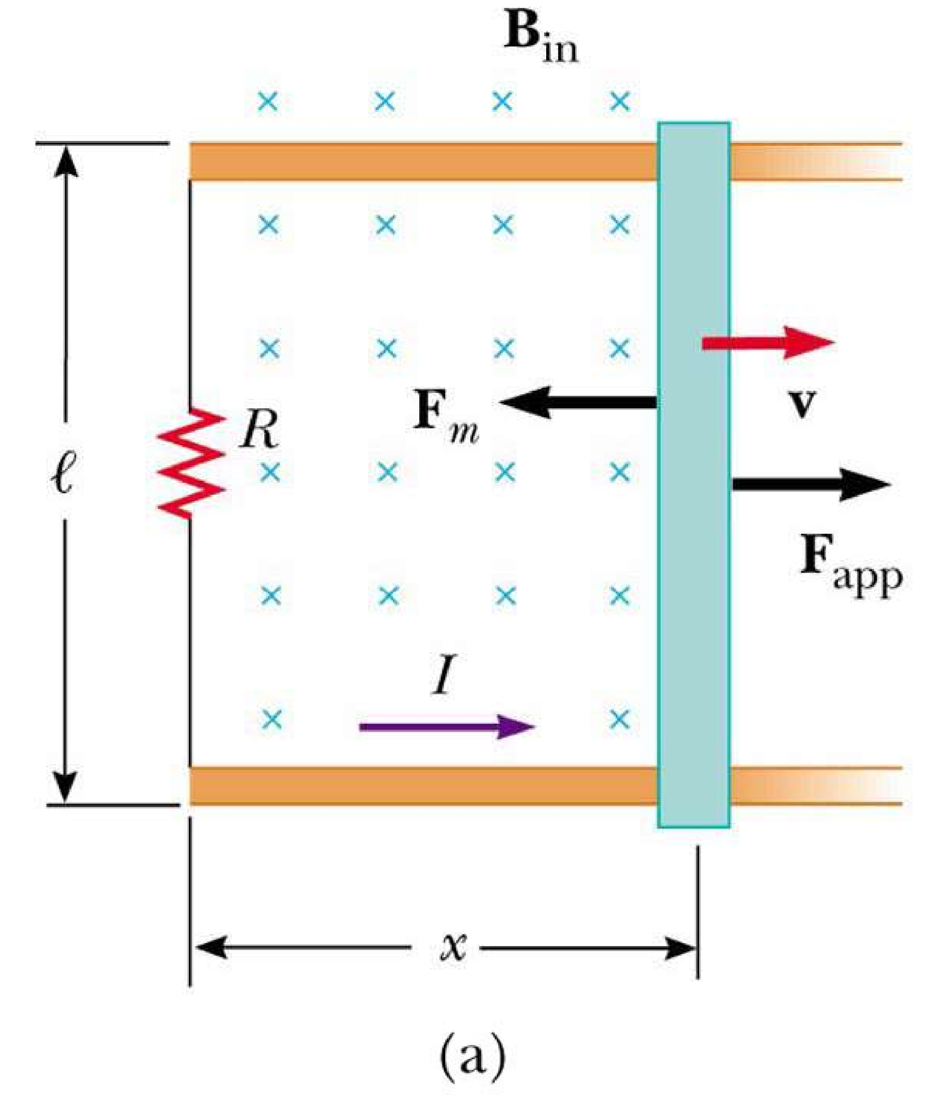
- As the negative charges accumulate at the base, a net positive charge exists at the upper end of the conductor
- As a result of this charge separation, an electric field is produced in the conductor
- Charges build up at the ends of the conductor until the downward magnetic force is balanced by the upward electric force
- There is a potential difference between the upper and lower ends of the conductor

Motional emf

- The potential difference between the ends of the conductor can be found by
  - $\Delta V = B \ell v$
  - The upper end is at a higher potential than the lower end
- A potential difference is maintained across the conductor as long as there is motion through the field
  - If the motion is reversed, the polarity of the potential difference is also reversed

Motional emf in a Circuit

- Assume the moving bar has zero resistance
- As the bar is pulled to the right with velocity v under the influence of an applied force, F, the free charges experience a magnetic force along the length of the bar
- This force sets up an induced current because the charges are free to move in the closed path

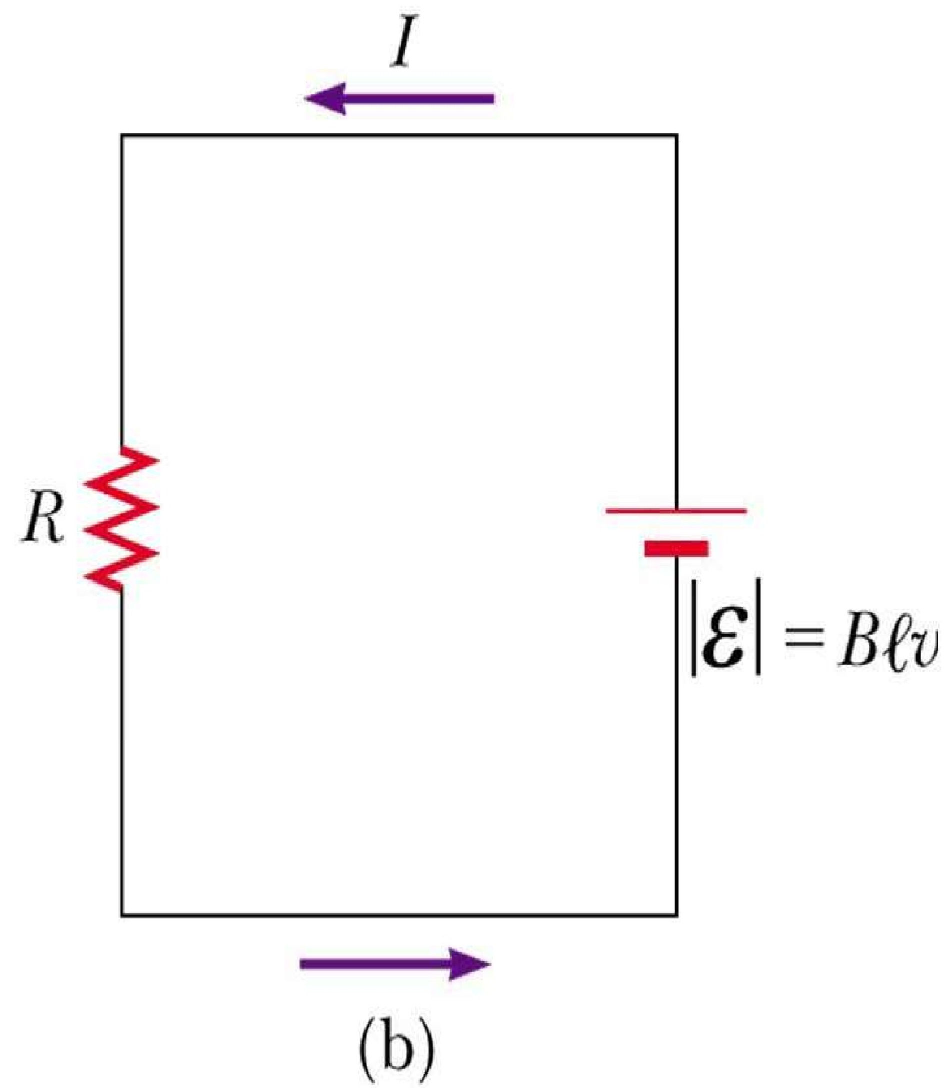


Motional emf in a Circuit

- The changing magnetic flux through the loop and the corresponding induced emf in the bar result from the *change in area* of the loop
- The induced, motional emf, acts like a battery in the circuit

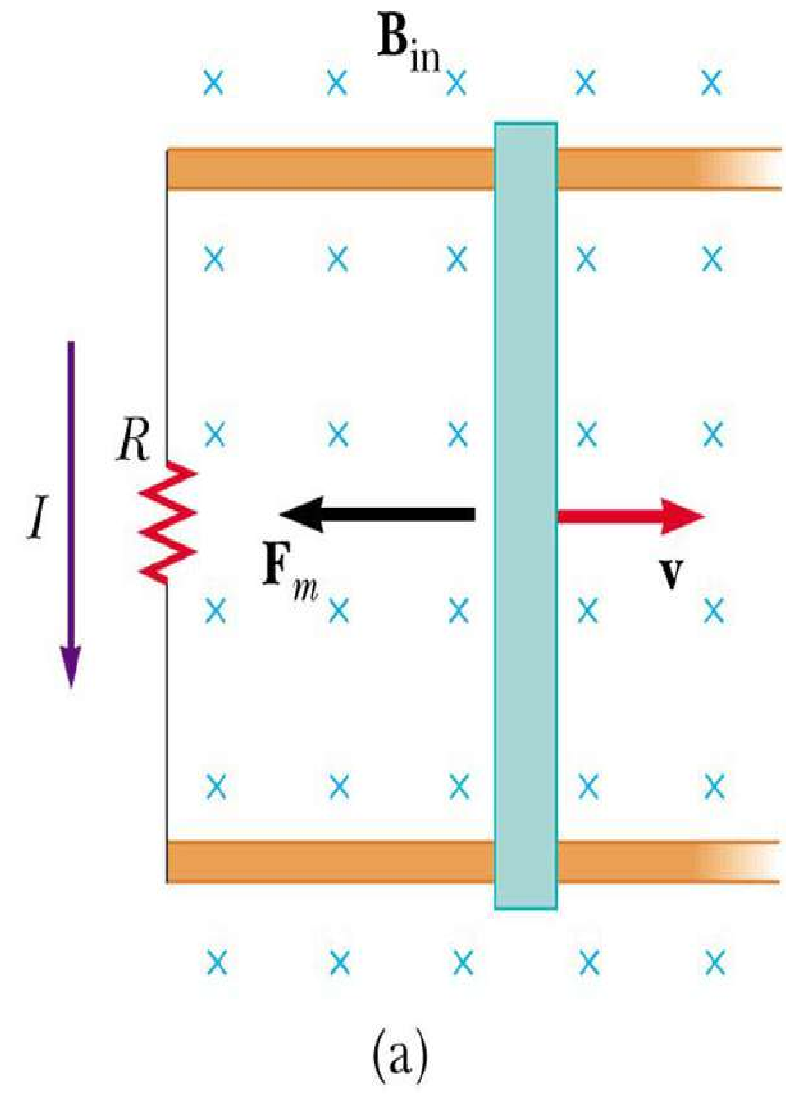
$$|\varepsilon| = B\ell v \text{ and}$$

$$I = \frac{B\ell v}{R}$$



Lenz' Law Revisited – Moving Bar Example

- As the bar moves to the right, the magnetic flux through the circuit increases with time because the area of the loop increases
- The induced current must be in a direction such that it opposes the change in the external magnetic flux

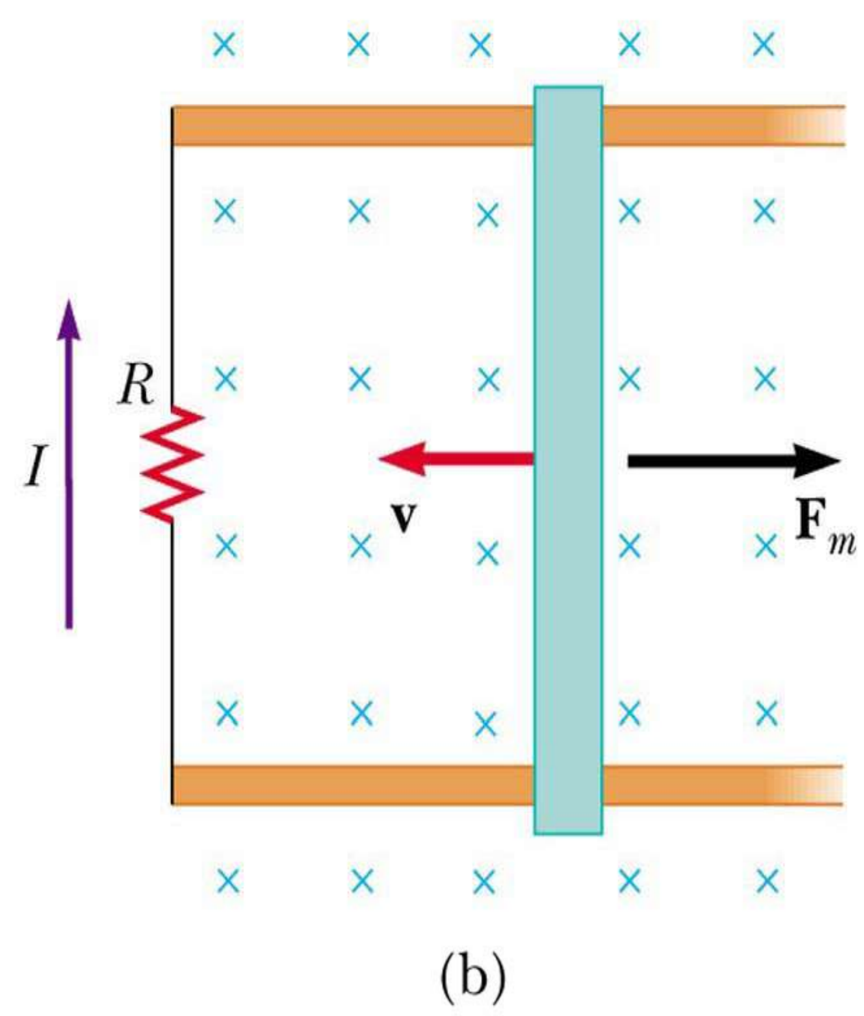


Lenz' Law, Bar Example

- The flux due to the external field in increasing into the page
- The flux due to the induced current must be out of the page
- Therefore the current must be counterclockwise when the bar moves to the right

Lenz' Law, Bar Example, final

- The bar is moving toward the left
- The magnetic flux through the loop is decreasing with time
- The induced current must be clockwise to produce its own flux into the page

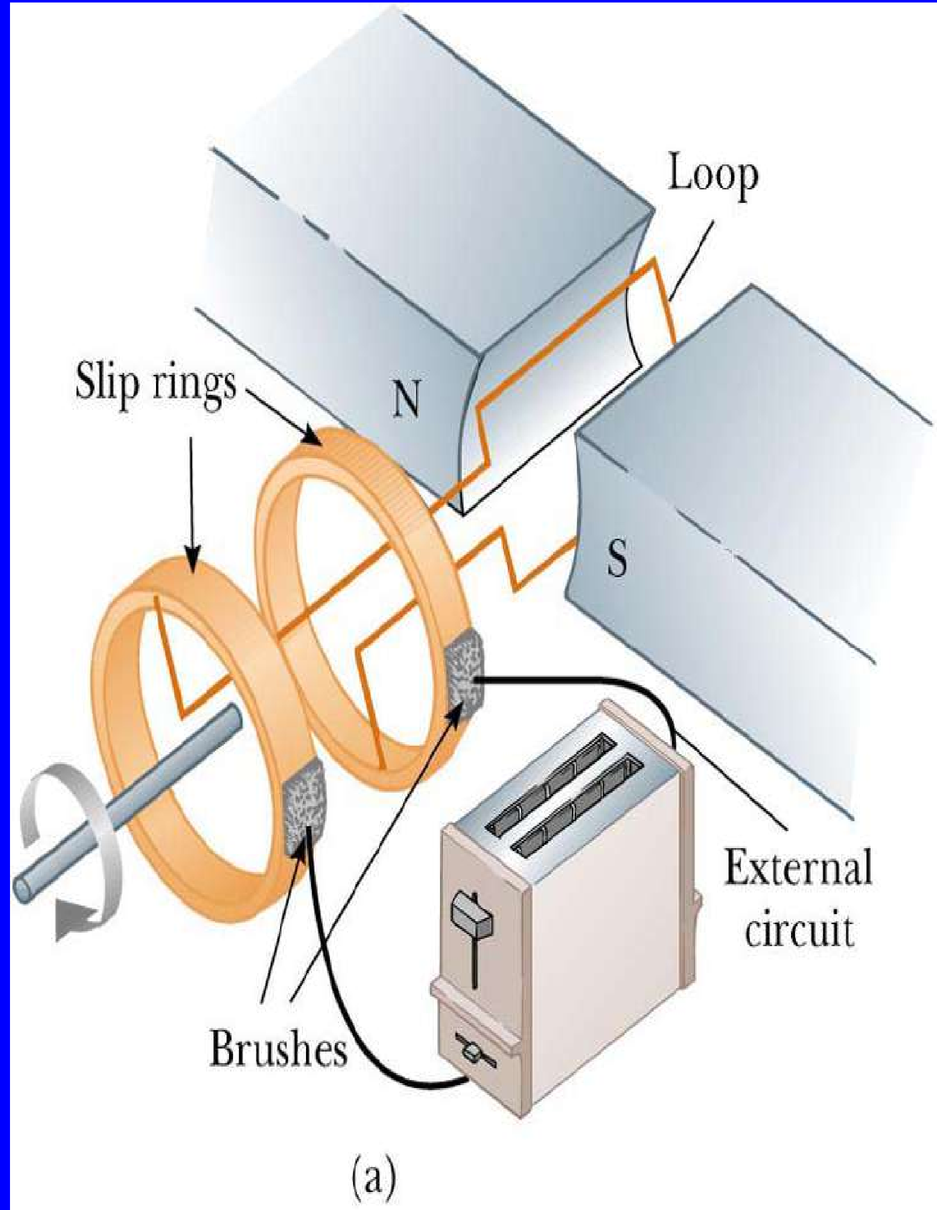


Generators

- Alternating Current (AC) generator
  - Converts mechanical energy to electrical energy
  - Consists of a wire loop rotated by some external means
  - There are a variety of sources that can supply the energy to rotate the loop
    - These may include falling water, heat by burning coal to produce steam

AC Generators

- Basic operation of the generator
  - As the loop rotates, the magnetic flux through it changes with time
  - This induces an emf and a current in the external circuit
  - The ends of the loop are connected to slip rings that rotate with the loop
  - Connections to the external circuit are made by stationary brushed in contact with the slip rings



AC Generators

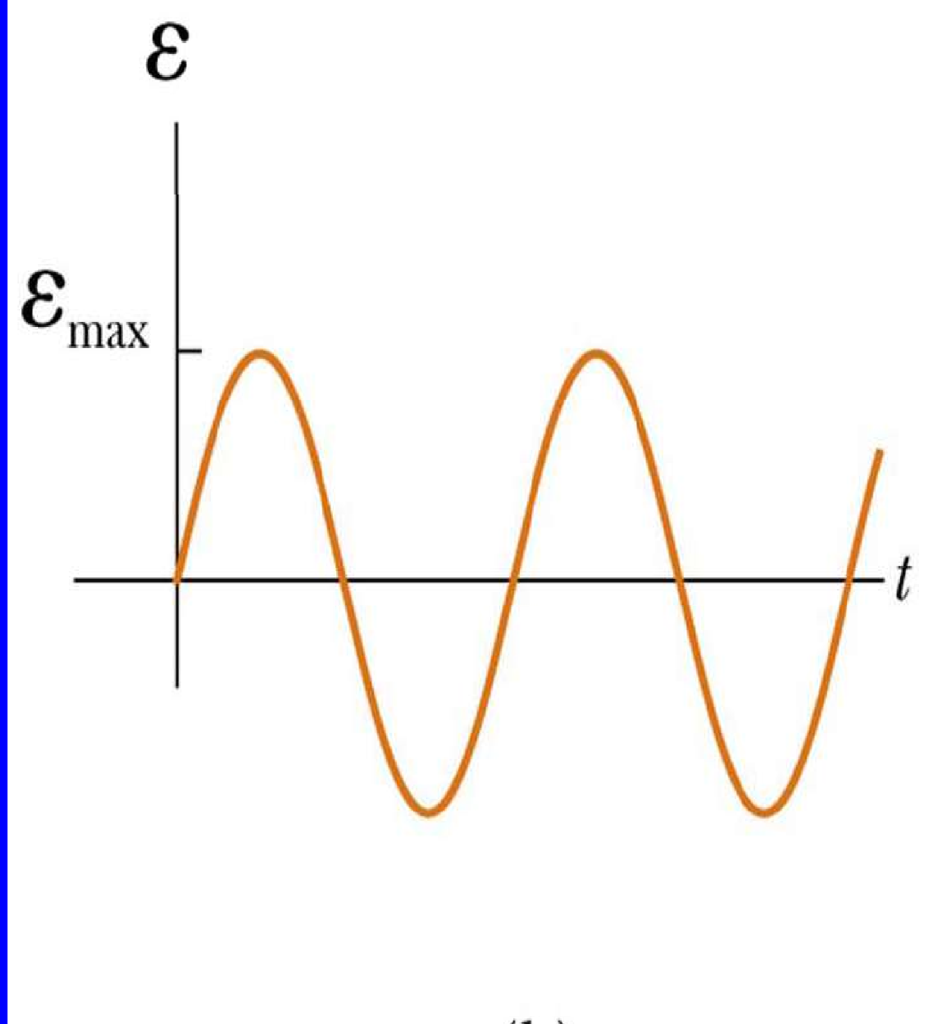
- The emf generated by the rotating loop can be found by

  $\varepsilon = 2 B \ell v_{\perp} = 2 B \ell \sin \theta$

- If the loop rotates with a constant angular speed, ω, and N turns

  $\varepsilon = N B A \omega \sin \omega t$

- $\varepsilon = \varepsilon_{max}$ when loop is parallel to the field
- $\varepsilon = 0$ when when the loop is perpendicular to the field



(b)

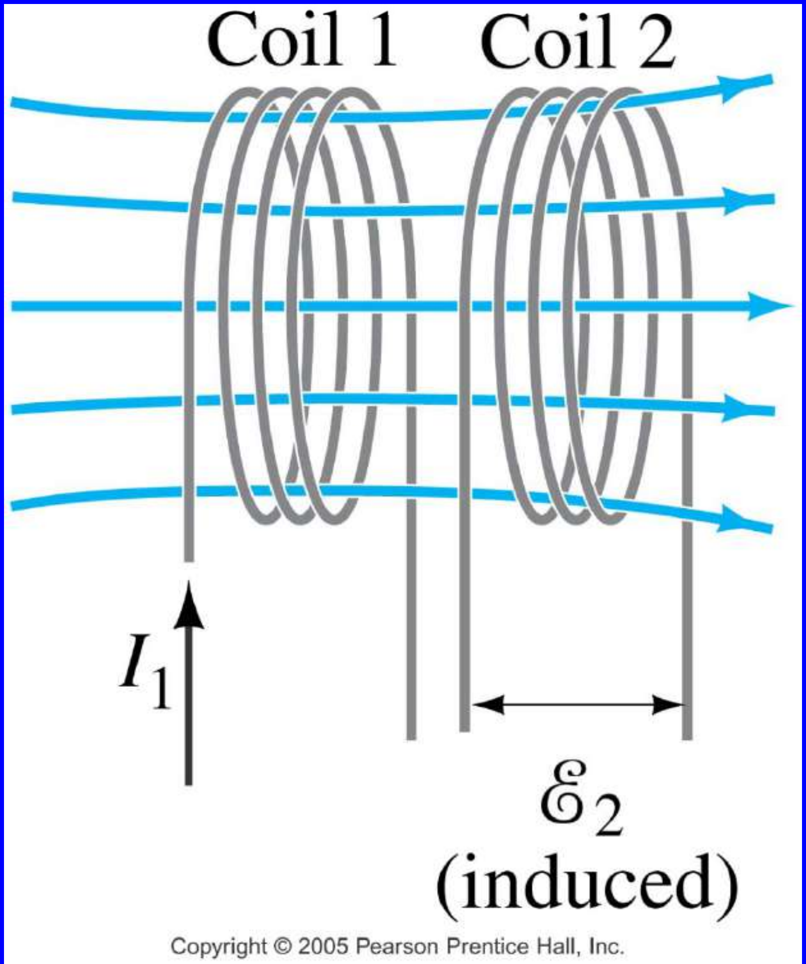
Inductance

Mutual inductance: a changing current in one coil will induce a current in a second coil.

$$\mathcal{E}_2 = -M\frac{\Delta I_1}{\Delta t}$$

And vice versa; note that the constant M, known as the mutual inductance, is the same:

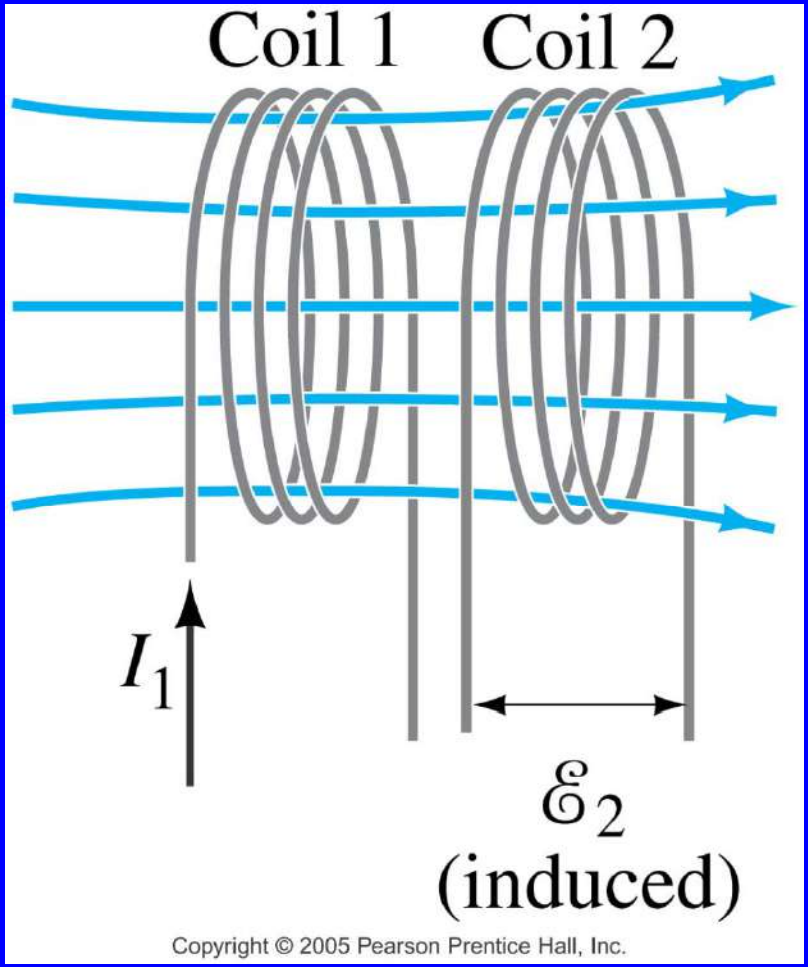
$$\mathcal{E}_1 = -M\frac{\Delta I_2}{\Delta t}$$



Unit of inductance: the henry, H.

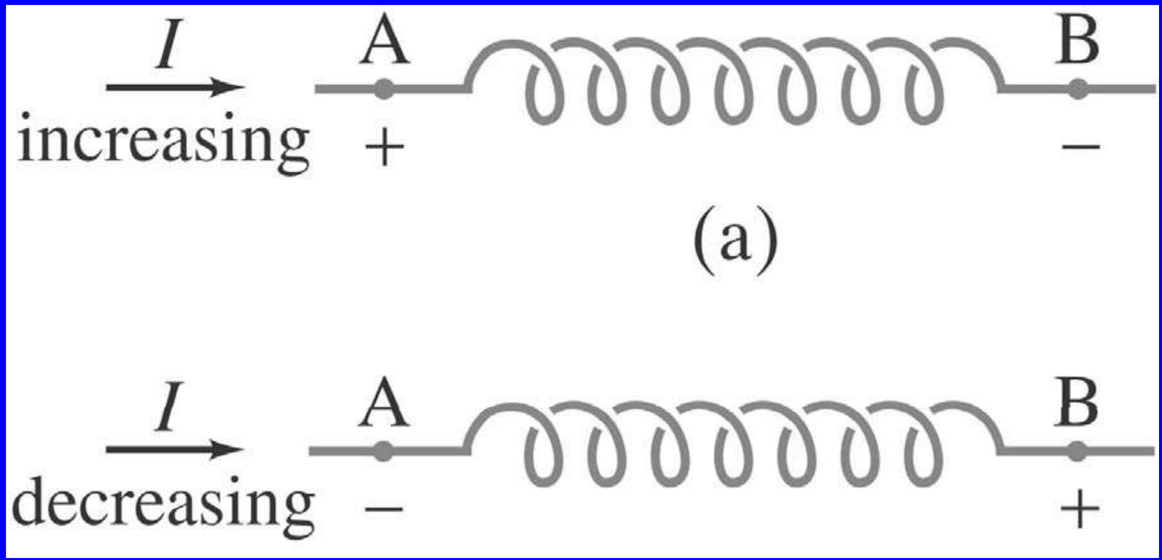
1 H = 1 V·s/A = 1 Ω·s.

A transformer is an example of mutual inductance.



Self-inductance

- *Self-inductance* occurs when the changing flux through a circuit arises from the circuit itself
  - As the current increases, the magnetic flux through a loop due to this current also increases
  - The increasing flux induces an emf that opposes the current



Self-inductance

- The self-induced emf must be proportional to the time rate of change of the current

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

  - L is a proportionality constant called the *inductance* of the device
  - The negative sign indicates that a changing current induces an emf in opposition to that change

Self-inductance

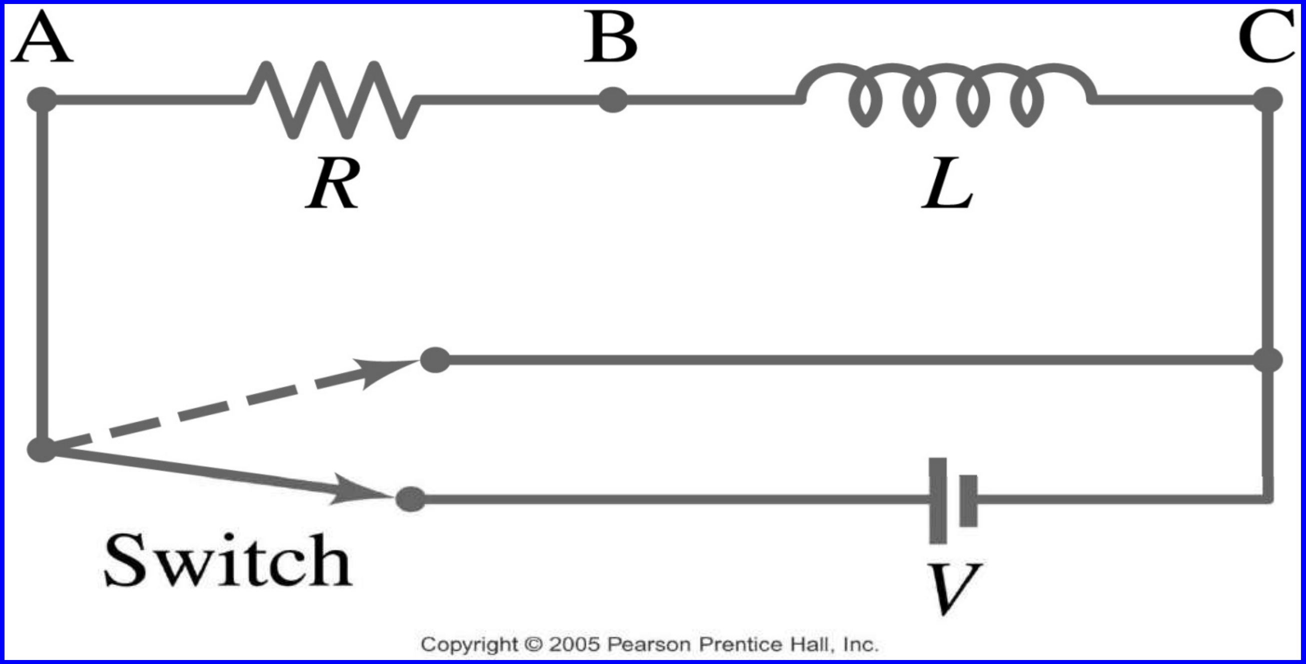
- The inductance of a coil depends on geometric factors
- The SI unit of self-inductance is the *Henry*
  - 1 H = 1 (V · s) / A
- You can determine an equation for L

$$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{NB}{I}$$

Inductor in a Circuit

- Inductance can be interpreted as a measure of opposition to the rate of change in the current
  - Remember resistance R is a measure of opposition to the current
- As a circuit is completed, the current begins to increase, but the inductor produces an emf that opposes the increasing current
  - Therefore, the current doesn't change from 0 to its maximum instantaneously

Inductor in a Circuit: *LR* Circuit



As a circuit is completed, the current begins to increase, but the inductor produces an emf that opposes the increasing current. Therefore, the current doesn't change from 0 to its maximum instantaneously.

Initially most of the voltage drop across the inductor, as the current is changing rapidly. With time, the current will increase less and less, until all the voltage is across the resistor.

The sum of potential differences around the loop gives

$$L\frac{dI}{dt} + RI = V_0.$$

Integrating gives the current as a function of time:

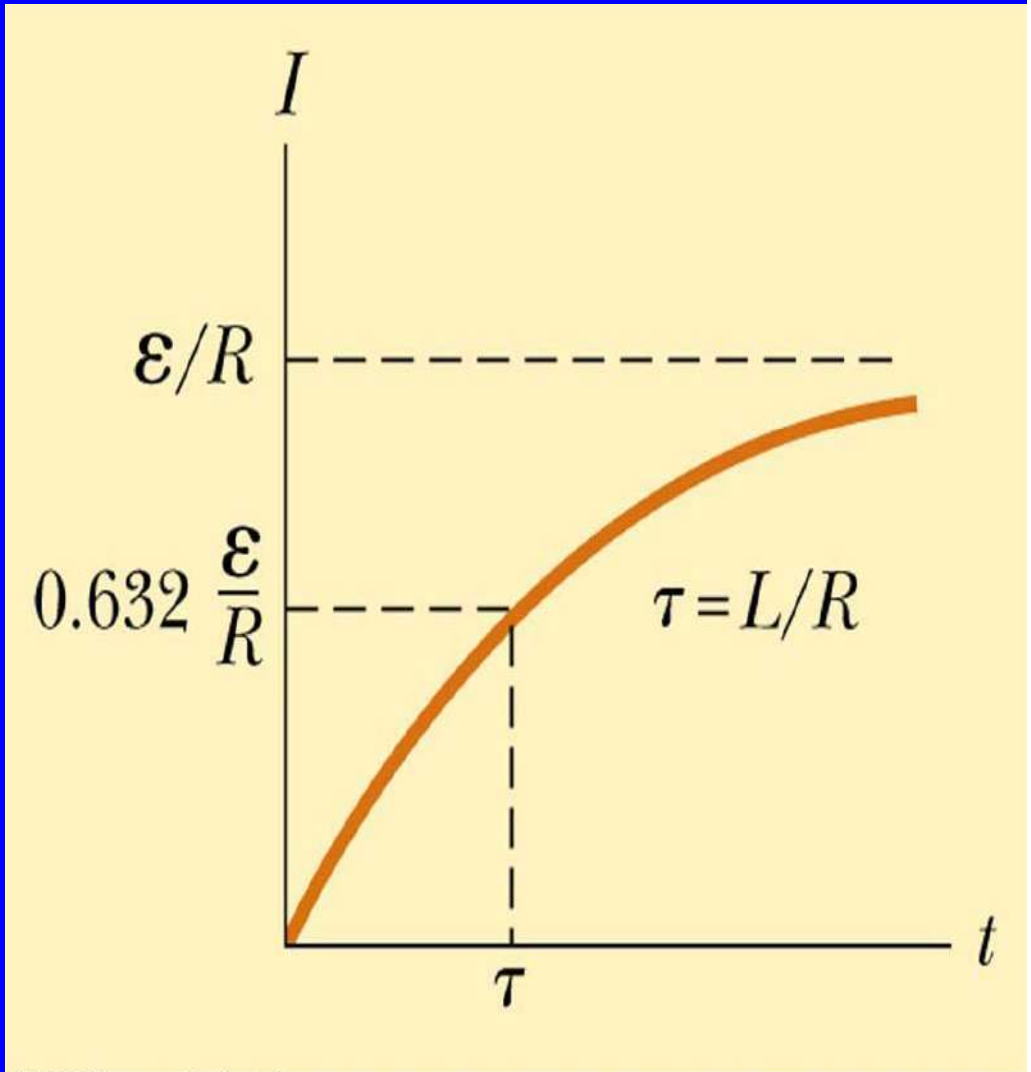
$$I = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right)$$

The time constant of an *LR* circuit is

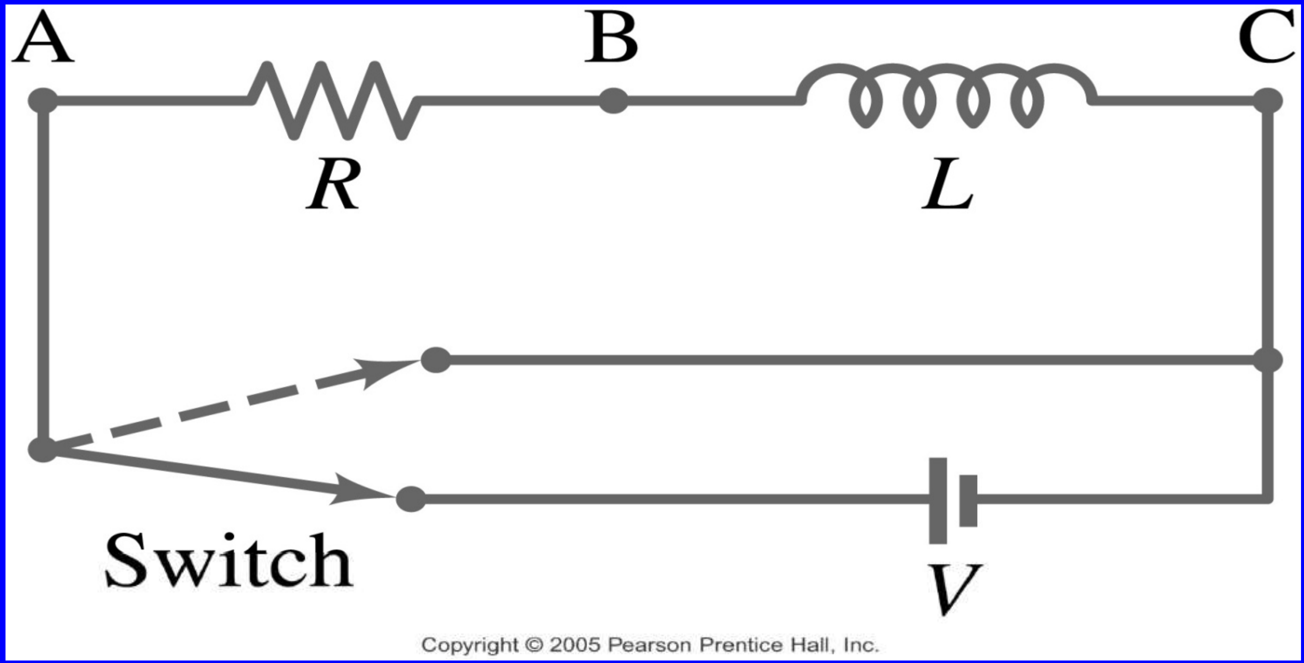
$$\tau = \frac{L}{R}$$

LR Circuit

- When the current reaches its maximum, the rate of change and the back emf are zero
- The time constant, τ, for an *LR* circuit is the time required for the current in the circuit to reach 63.2% of its final value
- The time constant τ depends on *L* and *R*



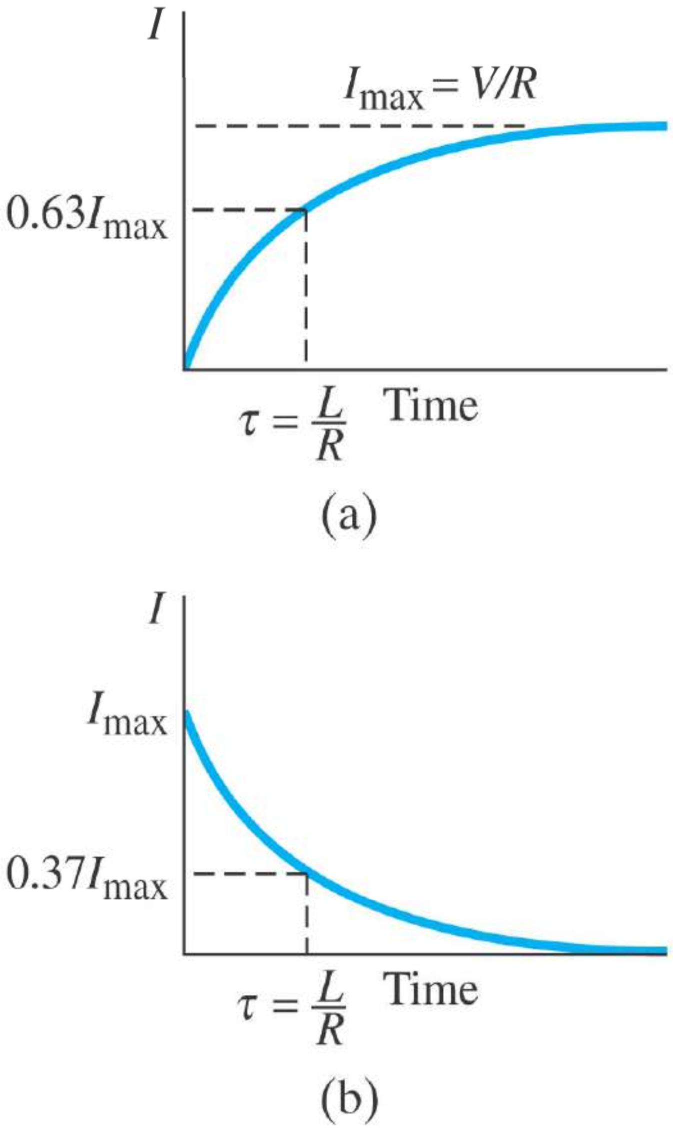
If the circuit is then shorted across the battery, the current will gradually decay away.



$$I = \left(\frac{V}{R}\right)\left(1 - e^{-t/\tau}\right)$$

$$I = I_{\max} e^{-t/\tau}$$

where $\tau = L/R$



Energy Stored in a Magnetic Field

- The emf induced by an inductor prevents a battery from establishing an instantaneous current in a circuit
- The battery has to do work to produce a current
  - This work can be thought of as energy stored by the inductor in its magnetic field
  - $PE_L = \frac{1}{2} L I^2$

Energy Stored in a Magnetic Field

Just as we saw that energy can be stored in an electric field, energy can be stored in a magnetic field as well, in an inductor, for example.

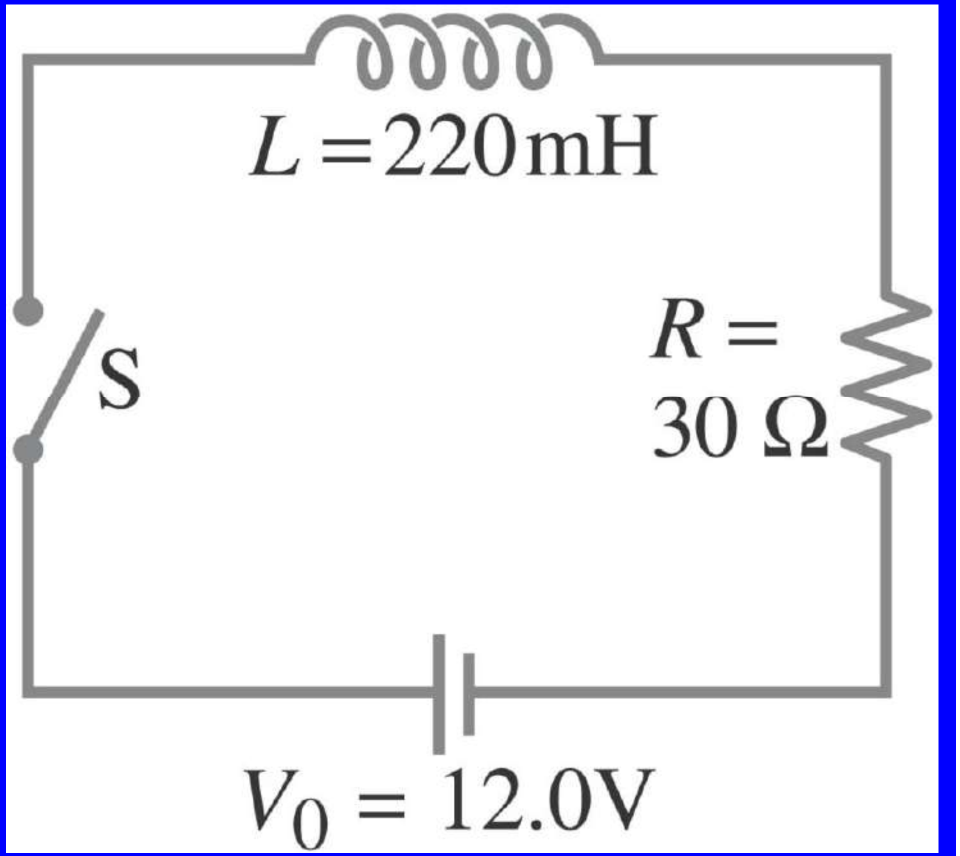
Analysis shows that the energy density of the field is given by:

$$u = \text{energy density} = \frac{1}{2}\frac{B^2}{\mu_0}$$

An *LR* circuit

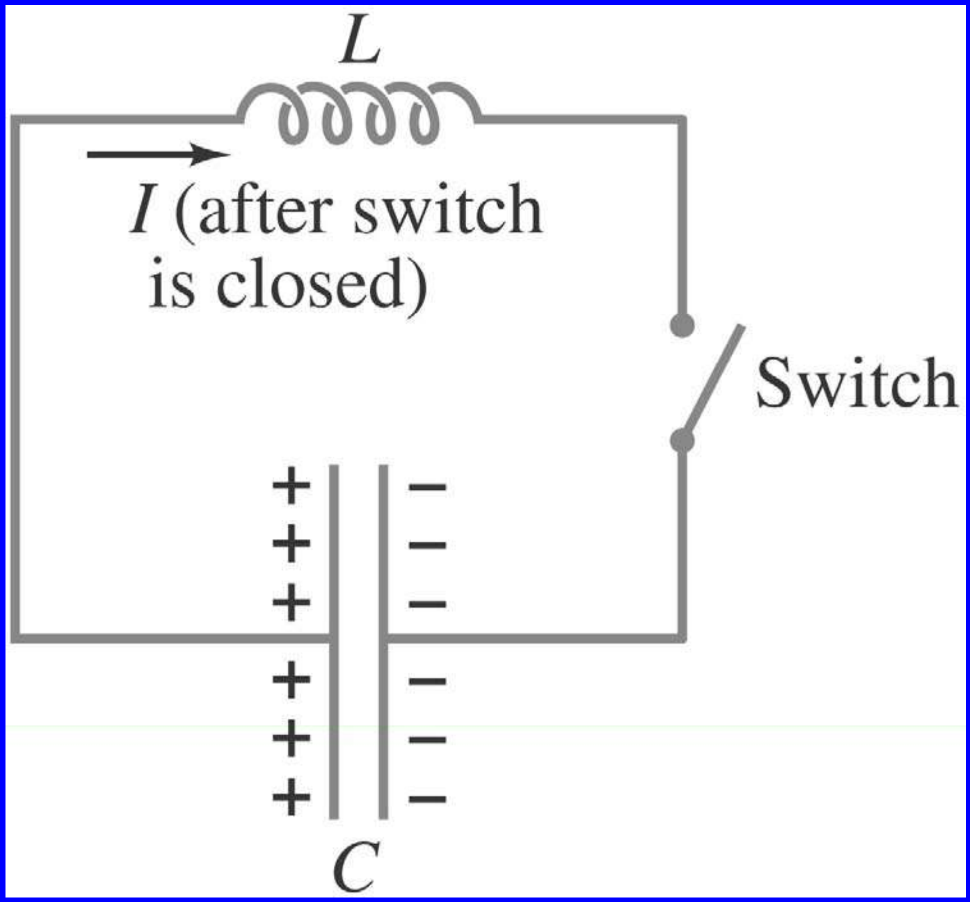
At $t = 0$, a 12.0-V battery is connected in series with a 220-mH inductor and a total of 30-Ω resistance, as shown.

(a) What is the current at $t = 0$?
(b) What is the time constant?
(c) What is the maximum current?
(d) How long will it take the current to reach half its maximum possible value?
(e) At this instant, at what rate is energy being delivered by the battery?



LC Circuits and Electromagnetic Oscillations

An *LC* circuit is a charged capacitor shorted through an inductor.



Summing the potential drops around the circuit gives a differential equation for Q:

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0.$$

This is the equation for simple harmonic motion, and has solutions

$$Q = Q_0 \cos(\omega t + \phi).$$

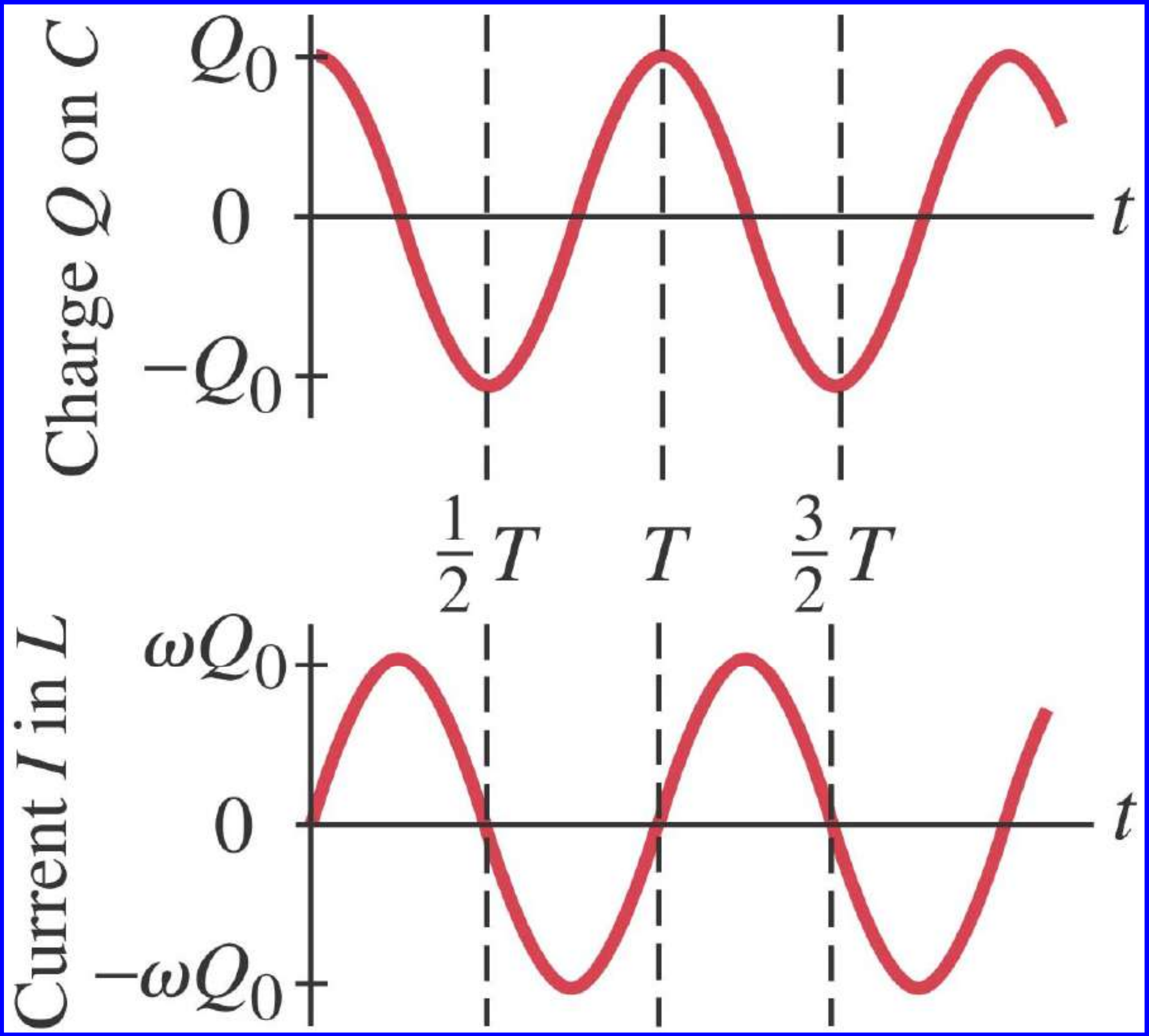
Substituting shows that the equation can only be true for all times if the frequency is given by

$$\omega = 2\pi f = \sqrt{\frac{1}{LC}}.$$

The current is sinusoidal as well:

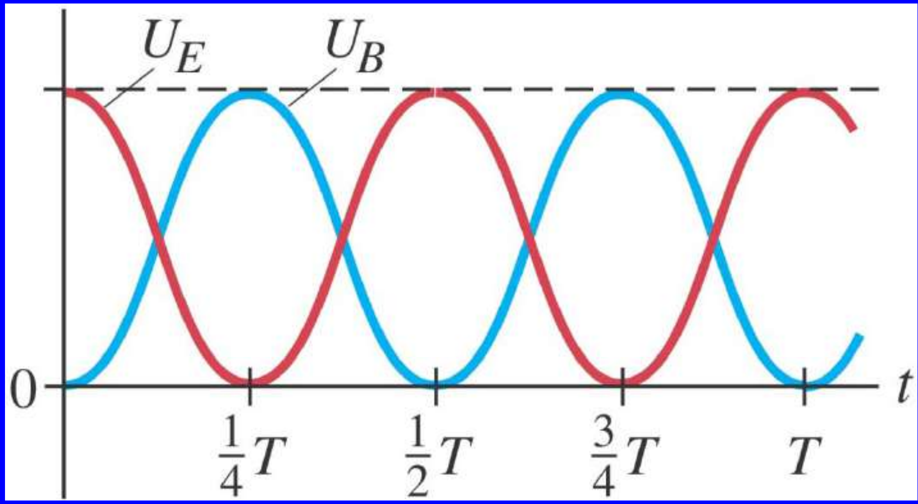
$$I = -\frac{dQ}{dt} = \omega Q_0 \sin(\omega t + \phi)$$
$$= I_0 \sin(\omega t + \phi).$$

The charge and current are both sinusoidal, but with different phases.

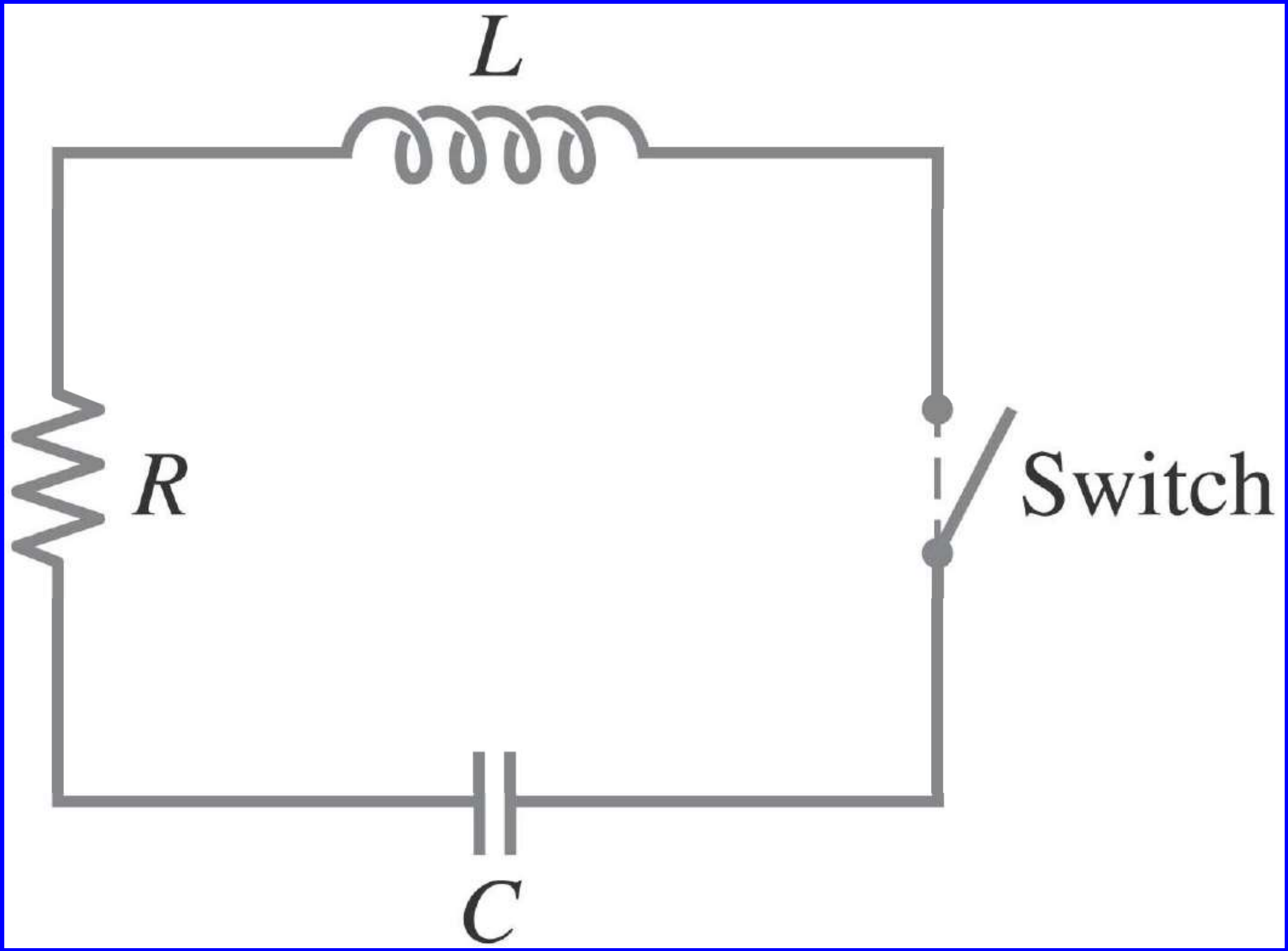


The total energy in the circuit is constant; it oscillates between the capacitor and the inductor:

$$U = U_E + U_B = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$
$$= \frac{Q_0^2}{2C}\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right] = \frac{Q_0^2}{2C}.$$



Any real (nonsuperconducting) circuit will have resistance.

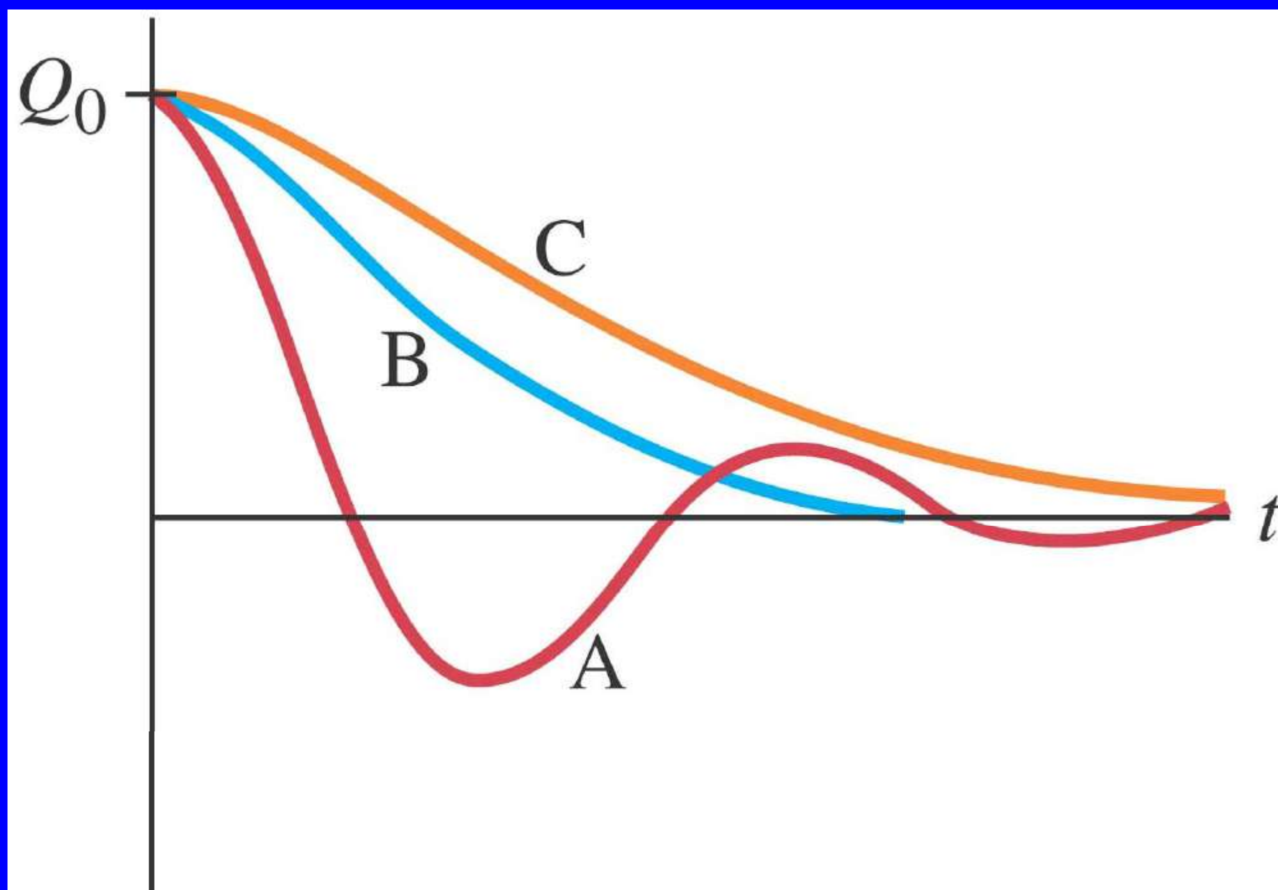


Now the voltage drops around the circuit give

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0.$$

The solutions to this equation are damped harmonic oscillations. The system will be underdamped for $R^2 < 4L/C$, and overdamped for $R^2 > 4L/C$. Critical damping will occur when $R^2 = 4L/C$.

This figure shows the three cases of underdamping, overdamping, and critical damping.



The angular frequency for underdamped oscillations is given by

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

The charge in the circuit as a function of time is

$$Q = Q_0 e^{-\frac{R}{2L}t} \cos(\omega' t + \phi)$$

Damped oscillations

At t = 0, a 40-mH inductor is placed in series with a resistance R = 3.0 Ω and a charged capacitor C = 4.8 μF. (a) Show that this circuit will oscillate. (b) Determine the frequency. (c) What is the time required for the charge amplitude to drop to half its starting value? (d) What value of R will make the circuit nonoscillating?