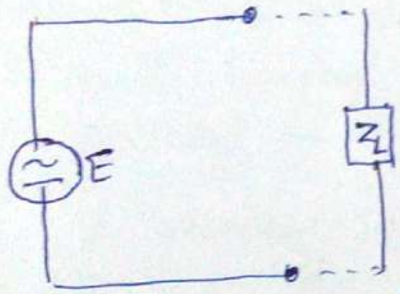
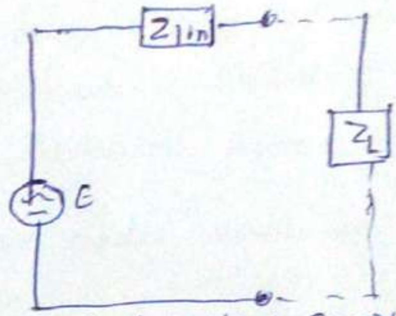
B.sc II nd Sem. (CBSC) & B.sc VI th Sem. paper 6.2

Network Analysis and Network Theorems

Any electrical circuit containing resistances, capacitances, inductances and generators (i.e. source) is known as electrical network. The circuit elements are of two types a) passive and b) active. The inductor resistor and capacitor with two terminals are regarded as passive elements as they are not fundamental source of power. The active elements are those which feed the power in to the given circuits. These are of two types a) voltage source and b) current source.



a) Ideal voltage Generator (a)

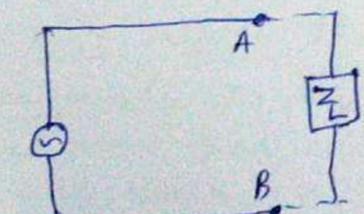


b) Real voltage source

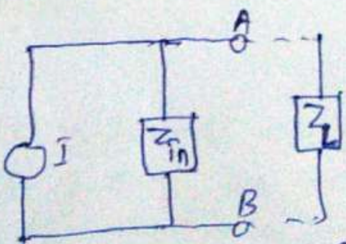
An ideal voltage source is an active device which can maintain a constant voltage across its terminals irrespective of the current supplied by it. It is also called constant voltage source. It is always equal to the open circuit voltage (above circuits, fig a)

A real voltage source has a finite internal impedance therefore the potential difference across its terminals decreases with increase of load impedance (fig b)

A constant current source is an active device which can supply the constant current to any load resistance that is connected across its terminals



a) Ideal current source

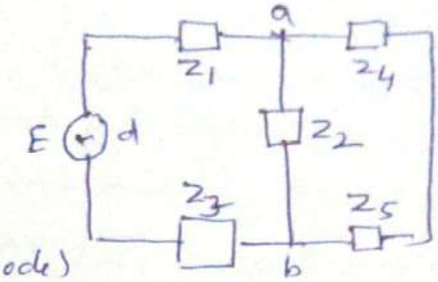


b) Real current source.

A Real source is a power sourse which contain a finite internal parallel Impedence (fig. b))

Networks - Some Important Definitions

From the Network



Junction (Node).! -
The point where two or more branches meet is called a junction (or node)
a and b are junctions

Branch! In a network any group of series elements having two terminals is called a branch The current in a branch remains same at each point.

An element joining two nodes, such as a and b. in fig. is called a branch.

A set of branches forming a closed path in a net work is called mesh.

Loop!- Any closed path in a network is called a loop. acba and abda are loops.

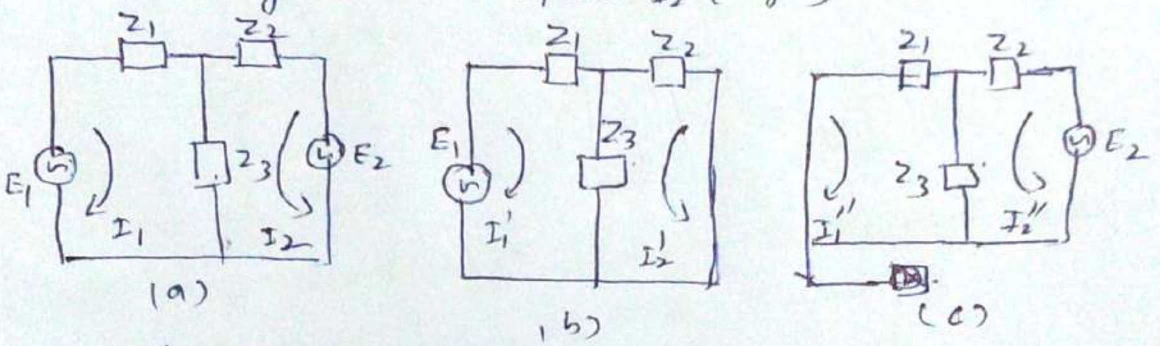
Active and passive Networks !- The networks containing sources of emf are called active networks, Networks containing no sources of emf are called passive networks

Linear and Non-Linear Networks ! - The network is said to be a linear. if it consists of linear impedences A linear impedence is any impedence that obey's ohm's law; In non-linear network, the relation voltage and current will be non-linear

Net works Theorem

statement!- In any linear network containing impedences and more than one Sourse of emf, the current flowing in any element is equal to algebraic Sum of the currents that will separately flow in that element if each source of emf. were considered seperately all the other sources being replaced at that time by their internal impedences.

proof! - Let us consider the network with two sources of emf's E_1 and E_2 with internal impedences z_1 and z_2 respectively,(fig) Let the current due to E_1 and E_2 acting together be I_1 and I_2 (fig a), Let current due to emf E_1 acting alone be I_1' and I_2' (fig b) Let the current due to E_2 acting alone be I_1'' and I_2'' (fig c)



Applying Kirchhoff's II law to fig (a) the mesh equations are

$$E_1 = (Z_1 + Z_3) I_1 + Z_3 I_2 \quad \text{———} (1)$$

$$E_2 = Z_3 I_1 + (Z_2 + Z_3) I_2 \quad \text{———} (2)$$

when E_1 is considered to act alone fig(b) the mesh equations are

$$E_1 = (Z_1 + Z_3) I_1' + Z_3 I_2' \quad \text{—} (3)$$

$$0 = Z_3 I_1' + (Z_2 + Z_3) I_2' \quad \text{—} (4)$$

when E_2 is considered to act alone fig(c) the mesh equations are

$$0 = (Z_1 + Z_3) I_1'' + Z_3 I_2'' \quad \text{—} (5)$$

$$E_2 = Z_3 I_1'' + (Z_2 + Z_3) I_2'' \quad \text{—} (6)$$

Adding Eqn (3) and (5)

$$E_1 = (Z_1 + Z_3)(I_1' + I_1'') + Z_3(I_2' + I_2'') \quad \text{—} (7)$$

Adding Eqn (4) & (6)

$$E_2 = (Z_2 + Z_3)(I_2' + I_2'') + Z_3(I_1' + I_1'') \quad \text{—} (8)$$

Eqn (7) and (8) will be identical with Eqn (1) and (2) if

$I_1 = I_1' + I_2''$ and $I_2 = I_2' + I_2''$

Thus the theorem proved.

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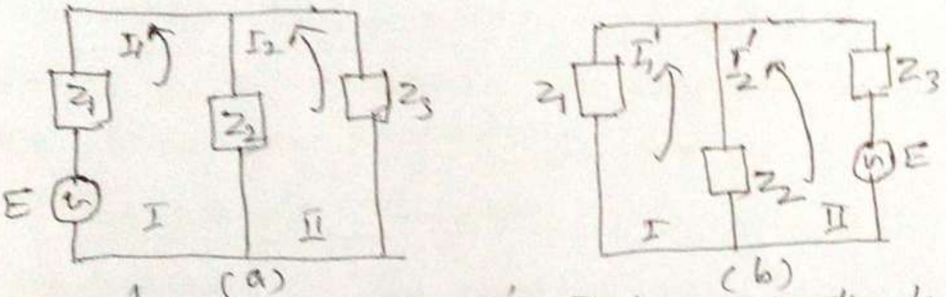
Reciprocity Theorem

Statement :- If an emf applied in one mesh of network of linear impedences produces a certain current in the second mesh, then the same emf acting in the second mesh will give an identical current in the first mesh

Proof! consider a network [fig(a)] Here the source of emf E is in the first mesh, Let the currents in the first and second meshes be I_1 and I_2 respectively.



Applying Kirchhoff's II Law to the two meshes

$$I_1(Z_1+Z_2) - I_2 Z_2 = E \quad \text{—— (1)}$$

and

$$I_2(Z_2+Z_3) - I_1 Z_2 = 0 \quad \text{—— (2)}$$

Substuting the value of I_1 from Eqn (2) in Eqn (1) we get

$$I_2\left[\frac{(Z_1+Z_2)(Z_2+Z_3)}{Z_2} - Z_2\right] = E$$

or $$I_2 = \frac{E \cdot Z_2}{(Z_1+Z_2)(Z_2+Z_3) - Z_2^2} \quad \text{—— (3)}$$

consider the network of [fig(b)]. Here the source of emf is in the second mesh, Let the current in the first and second meshes be I_1' & I_2' respectively.

Applying Kirchhoff's II Law to the two meshes we get

$$I_1'(Z_1+Z_2) - I_2' Z_2 = 0 \quad \text{—— (4)}$$

& $$I_2'(Z_2+Z_3) - I_1' Z_2 = E, \quad \text{—— (5)}$$

Substuting the value of I_2' from Eqn (4) in Eqn (5) we get

$$I_1'\left[\frac{(Z_1+Z_2)(Z_2+Z_3)}{Z_2}-Z_2\right]=E$$

or $$I_1' = \frac{E\cdot Z_2}{(Z_1+Z_2)(Z_2+Z_3)-Z_2^2} \quad \text{——} (6)$$

comparing Eqn (3) and Eq. (6) we have

$$I_2 = I_1'$$

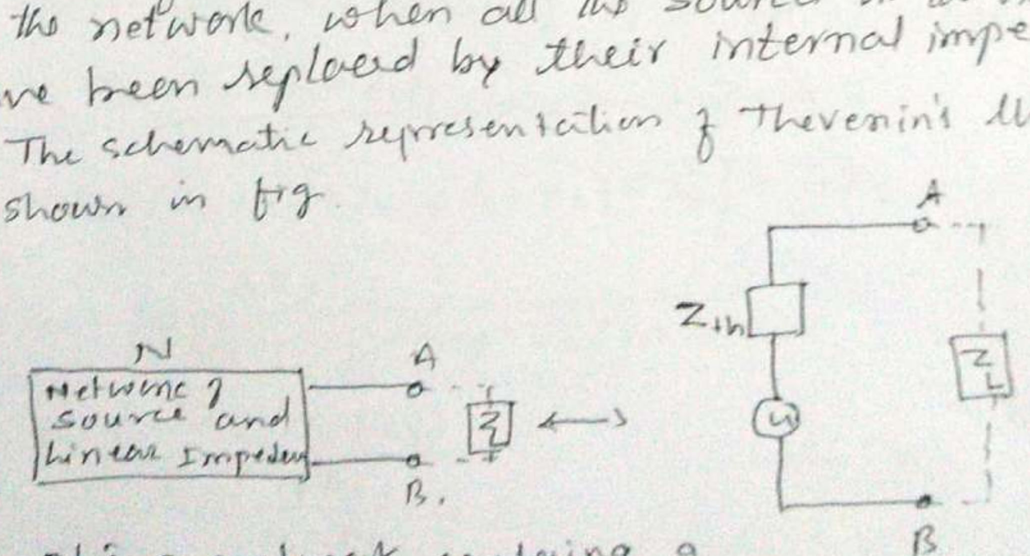
Thus the reciprocity theorem is proved.

__Tranfere__ __Impedence__:- The ratio of emf E in one mesh to the current I in another mesh is called the transfer Impedence Z_T

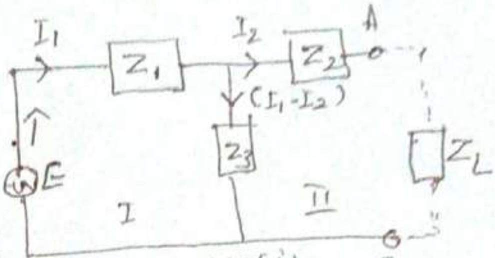
__Thevenin's Theorem__

__Statiment__:- The current in a load impedence connected between two terminals of a network of sources and linear impedences is same as due to a single voltage source of emf equal to open circuit voltage between the same terminals of the network and internal impedence equal to the impe-dence of the network between the same terminals of the network, when all the sources in the network have been replaced by their internal impedences

The schematic representation of Thevenin's theorem as shown in fig.



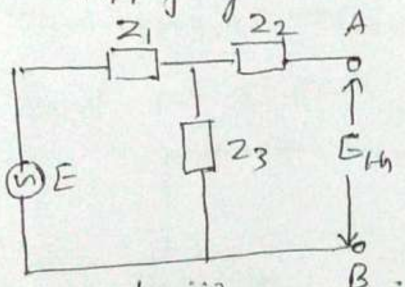
N is a network containing a number of sources and linear Impedences with output terminals A and B. Let E_{th} be the open-circuit voltage across A and B. Let Z_{th} be the impedence measured between the terminals A and B, when all the sources have been replaced by their respective internal impedence. The network N will produce the same current in an external load impedence Z_L connected across A and B as a single voltage source of emf E_{th} and internal impedence Z_{th} would do.



fig(i)

proof: consider a network containing resistive impedence Z_1, Z_2 and Z_3 and one source of emf E and internal impedence zero (i.e ideal source) (fig(ii) show)
I_1 is the current supplied by the source, I_2 is the current flowing through the load impedence Z_L

Applying Kirchhoff's law to the mesh I and II we get



fig(ii)

$$I_1 Z_1 + (I_1 - I_2) Z_3 = E$$

$$\text{or } I_1(Z_1+Z_2) - I_2 Z_3 = E \quad \text{---(1)}$$

$$\text{and } I_2 Z_2 + I_2 Z_L - (I_1 - I_2) Z_3 = 0$$

$$I_2(Z_2 + Z_L + Z_3 = I_1 Z_3 \quad \text{---(2)}$$

$$\text{or } I_1 = \frac{I_2(Z_2+Z_3+Z_L)}{Z_3}$$

substuting the value of I_1 in Eqn (1) we get

$$\frac{I_2(Z_2+Z_3+Z_L)}{Z_3}(Z_1+Z_2) - I_2 Z_3 = E$$

$$\text{or } I_2\left[(Z_2+Z_3+Z_L)(Z_1+Z_3) - Z_3^2\right] = E Z_3$$

$$\text{or } I_2 = \frac{E Z_3}{Z_2(Z_1+Z_3) + Z_1 Z_3 + Z_L(Z_1+Z_3)}$$

$$\therefore \; I_2 = \frac{E Z_3/(Z_1+Z_3)}{Z_1 + \left(\frac{Z_1 Z_3}{Z_1+Z_3}\right) + Z_L} \quad \text{---(3)}$$

Again from the fig(ii) the open circuit voltage at terminals A & B. (i.e. when Z_L is removed) is

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$$E_{th} = \frac{EZ_3}{Z_1+Z_3} \quad \text{—— (4)}$$

Further if E is replaced by its zero internal impedance fig(iii) the impedance of network between A and B is

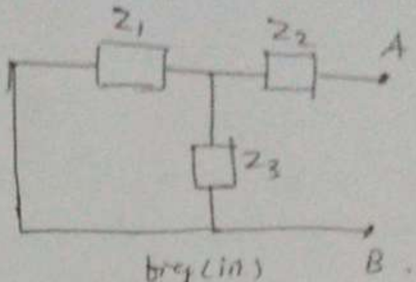


fig (iii)

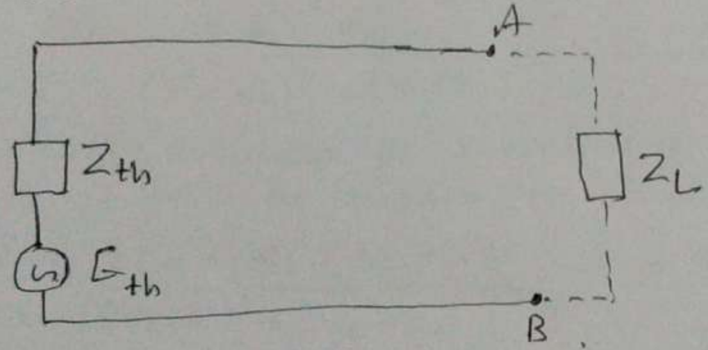
$$Z_{th} = Z_2 + \frac{1}{\frac{1}{Z_1}+\frac{1}{Z_3}}$$

$$= Z_2 + \frac{Z_1 Z_3}{Z_1+Z_3} \quad \text{—— (5)}$$

Thus Eqn (3) is equivalently written as.

$$I_2 = \frac{E_{th}}{Z_{th}+Z_L} \quad \text{—— (6)}$$

This proves Thevenin's theorem. The equivalent Circuit as shown below.



—x—

Maximum power Transfer theorem

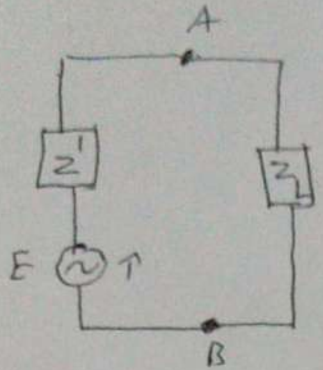
Statement :- The maximum power will be delivered to a load by a source when.

i) resistive component of the impedance of load and source are equal and.

ii) The reactive components of the impedances of source and load are equal in magnitude but opposite in sign

proof :- consider a voltage source of emf E' and complex internal impedance Z' and a load of.

complex impedance Z_L, connected across the source



Now we have

$$z' = R' + jx' \quad \text{——(1)}$$

and $Z_L = R_L + jX_L$ —(2)

Here R' and R_L are the resistive components and x' and X_L are the reactive components of Z' & Z_L respectively.

The current I flowing in the circuit is

$$I = \frac{E'}{Z' + Z_L}$$

or $$I = \frac{E'}{(R' + R_L) + j(x' + X_L)} \quad \text{——(3)}$$

The power delivered to the load is

$$P = I_2 R_L$$

or $$P = \frac{(E')^2 R_L}{(R' + R_L)^2 + (x' + X_L)^2} \quad \text{——(4)}$$

Now let us consider the variation of P with X_L.
The power P will be maximum when $(\partial P/\partial X_L) = 0$.

$$\frac{\partial P}{\partial X_L} = \frac{2(E')^2 R_L (X_L + x')}{[(R_L + R')^2 + (X_L + x')^2]^2} = 0.$$

or $$X_L = x' \quad \text{——(5)}$$

Thus if x' is inductive the X_L must be capacitive and of same magnitude of x'
when this condition is satisfied, the maximum power is

$$P_{max} = \frac{(E')^2 R_L}{(R_L + R')^2} \quad \text{——(6)}$$

Again consider the variation of P_{max} with R_L. The power P_{max} with R_L. The power P_{max} will be maximum then

$$\frac{\partial P_{max}}{\partial R_L} = \frac{(E')^2 (R_L + R')^2 - 2(E')^2 R_L (R_L + R')}{(R_L + R')^4} = 0,$$

$\therefore R_L = R'$ —(7) Eqn (7) shows that maximum power will be delivered to the load when the resistive component of source and load are equal. The ~~eqn~~ expression for the max. power may be written from Eqn (6) $P_{max} = \frac{E'^2 R_L}{(2R_L)^2} = (E')^2/4R_L$ (8)