Magnetism

Few words and mini CV of my self

1990 - 1995 M.sc.EE Student at IET, Aalborg University.
1995 - 1999 Ph.D. Student at IET, Aalborg University.
1999 - 2002 Assistant Professor at IET, Aalborg University.
2002 - Associate Professor at IET, Aalborg University.

Content of the presentation

- Maxwell's general equations on magnetism
- Hysteresis curve
- Inductance
- Magnetic circuit modelling
- Eddy currents and hysteresis losses
- Force / torque / power

Maxwell

James Clerk Maxwell 1831 - 1879

Unified theory of the connection between electricity and magnetism

Maxwells equations in free space

Not easy to understand in this form.
Practical engineers tend to forget the definition of divergence, curl, surface and line integral

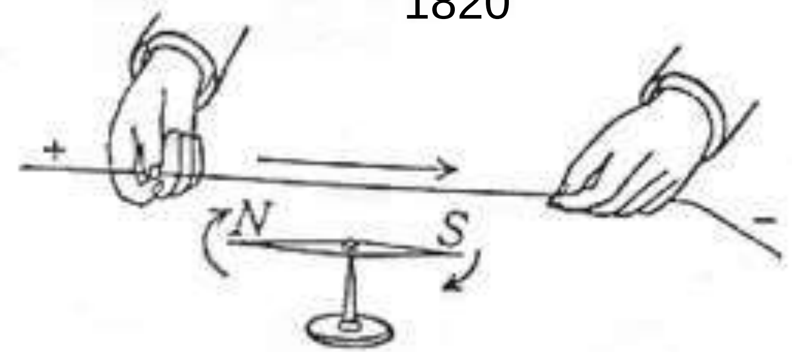
Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\oint_S \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q_S}{\varepsilon_0}$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Faraday's law of induction	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi_{B,S}}{\partial t}$
Ampère's circuital law with Maxwell's correction	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \frac{\partial \Phi_{E,S}}{\partial t}$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Ørsted

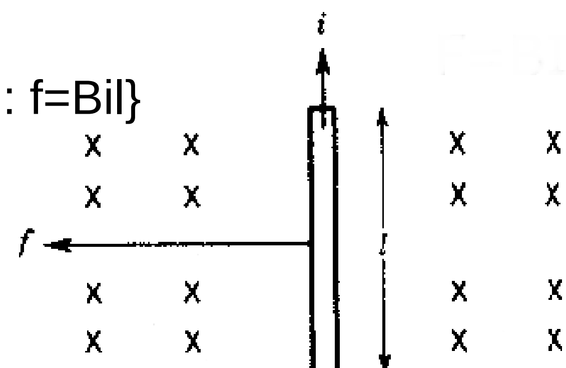
H.C Ørsted 1777-1851

1820



Ørsteds discovery was a deflection of a Compass needle when it is close to a current carrying conductor.
But he was not able to describe the physics or mathematics behind the phenomena

{f = Bxil / 90° : f=Bil}

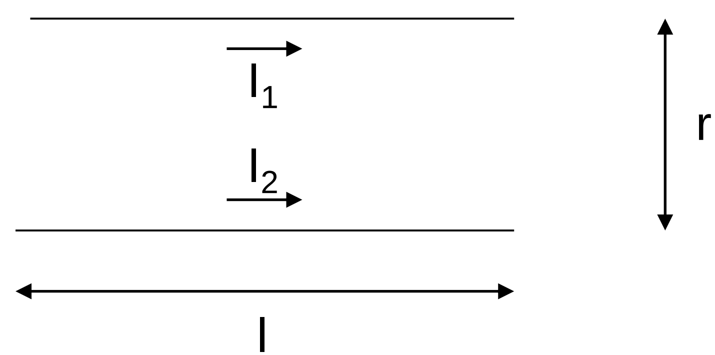


Amperé

André-Marie Ampére 1775 – 1836

Just one week after Ørsteds discovery Ampere showed that two current carrying wires attract or repulse each other, and was able to show that :

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 \cdot I_2}{r}$$



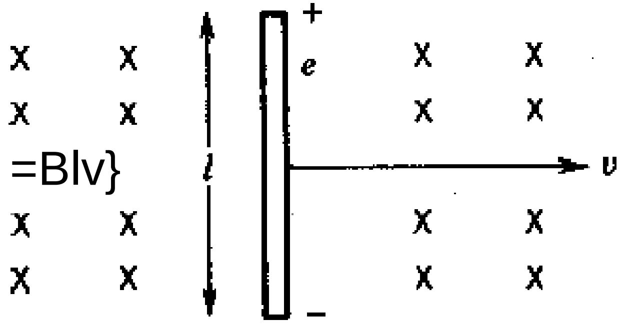
Thus, for two parallel wires carrying a current of 1 A, and spaced apart by 1 m in vacuum, the force on each wire per unit length is exactly 2 × 10^{-7} N/m.

It was first in 1826 he published the circuit law in Maxwell's equation

Faraday

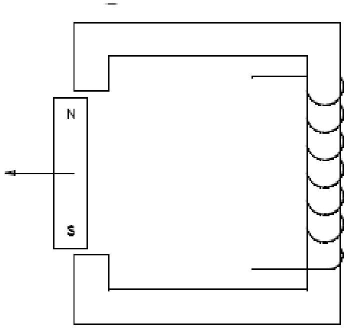
The Induction law 1821

Michael Faraday 1791-1867

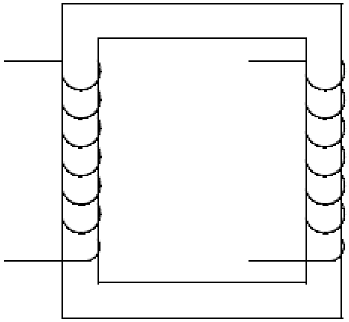


{e = Bxlv / 90° : e =Blv}

Induction by movement
”generator”



Induction by alternating current
”transformer”



Gauss

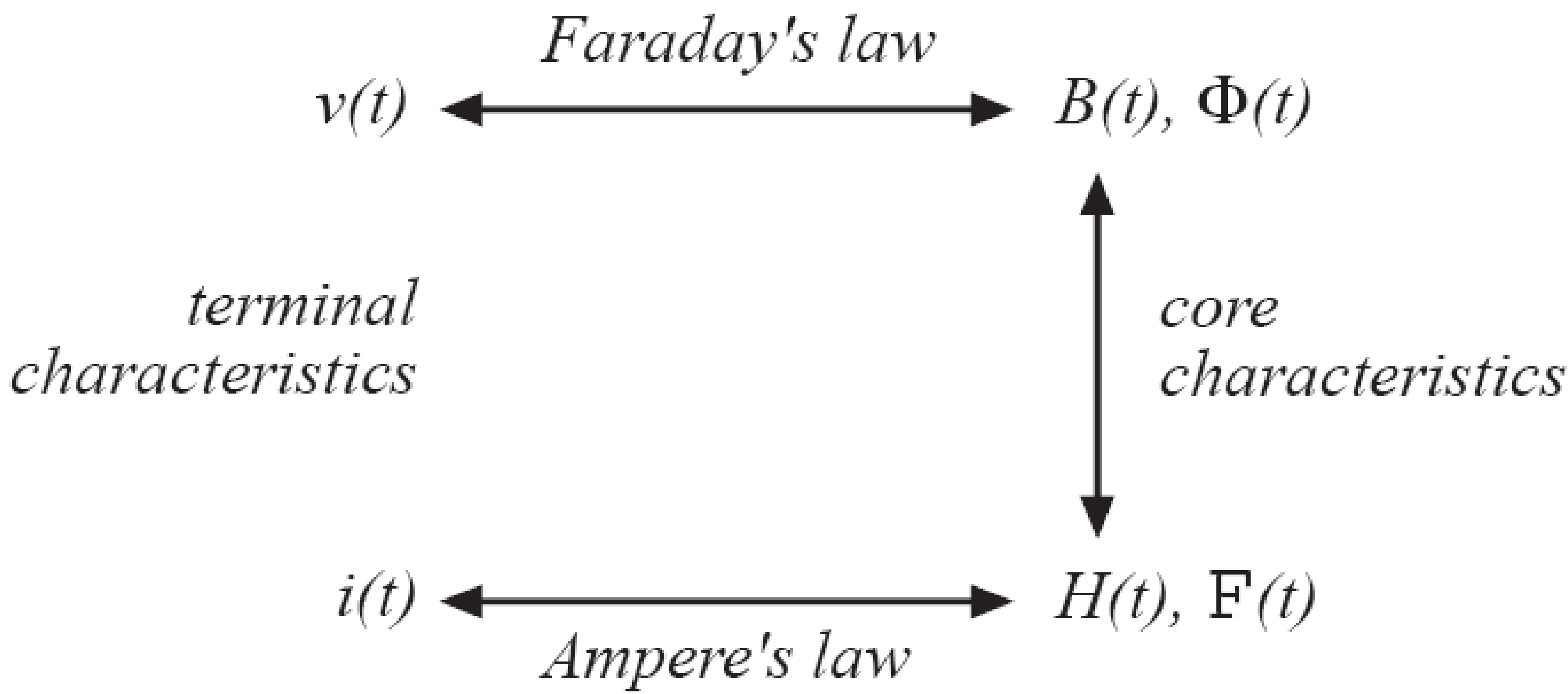
Carl Friedrich Gauss 1777 – 1855

$$\oint_S \mathbf{B} \cdot d\mathbf{A} = 0$$

Basically the magnetic induction can’t escape.
The equation says that we don’t have a monopole.
There will always be a north and a south pole

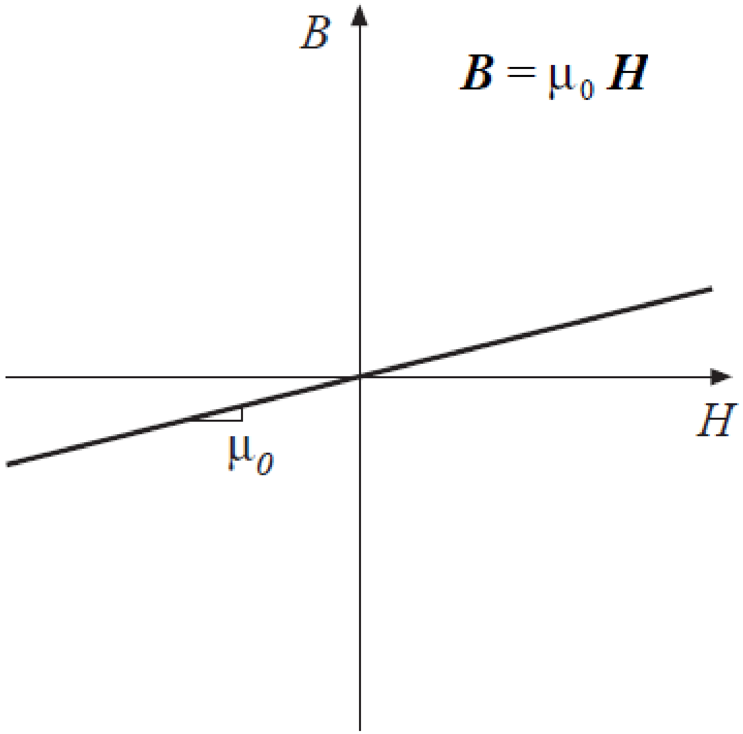
Relations in Maxwell's equations

Simplified analogies and the relations in Maxwell's equations

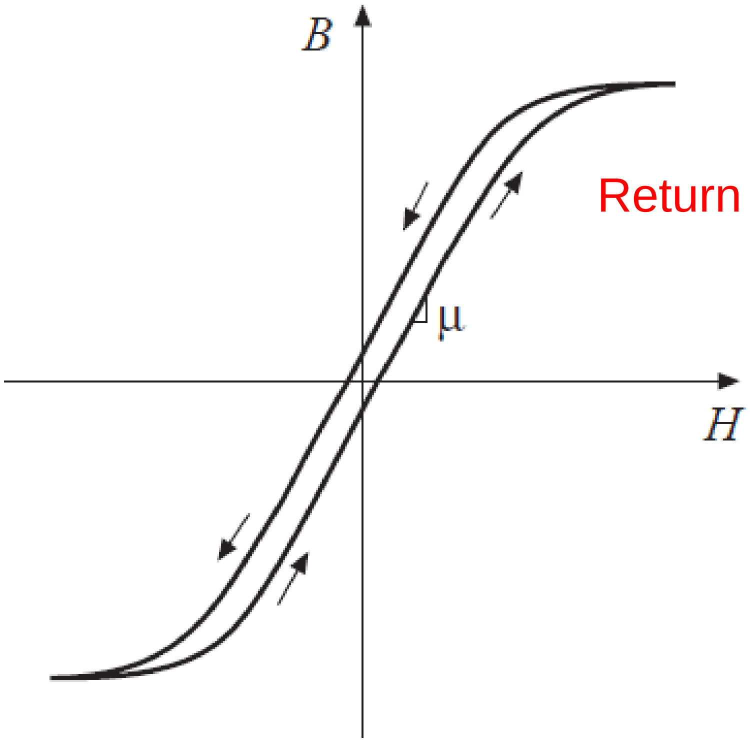


Permeability

Free space



A magnetic core material



μ_0 = permeability of free space

$= 4\pi \cdot 10^{-7}$ Henries per meter

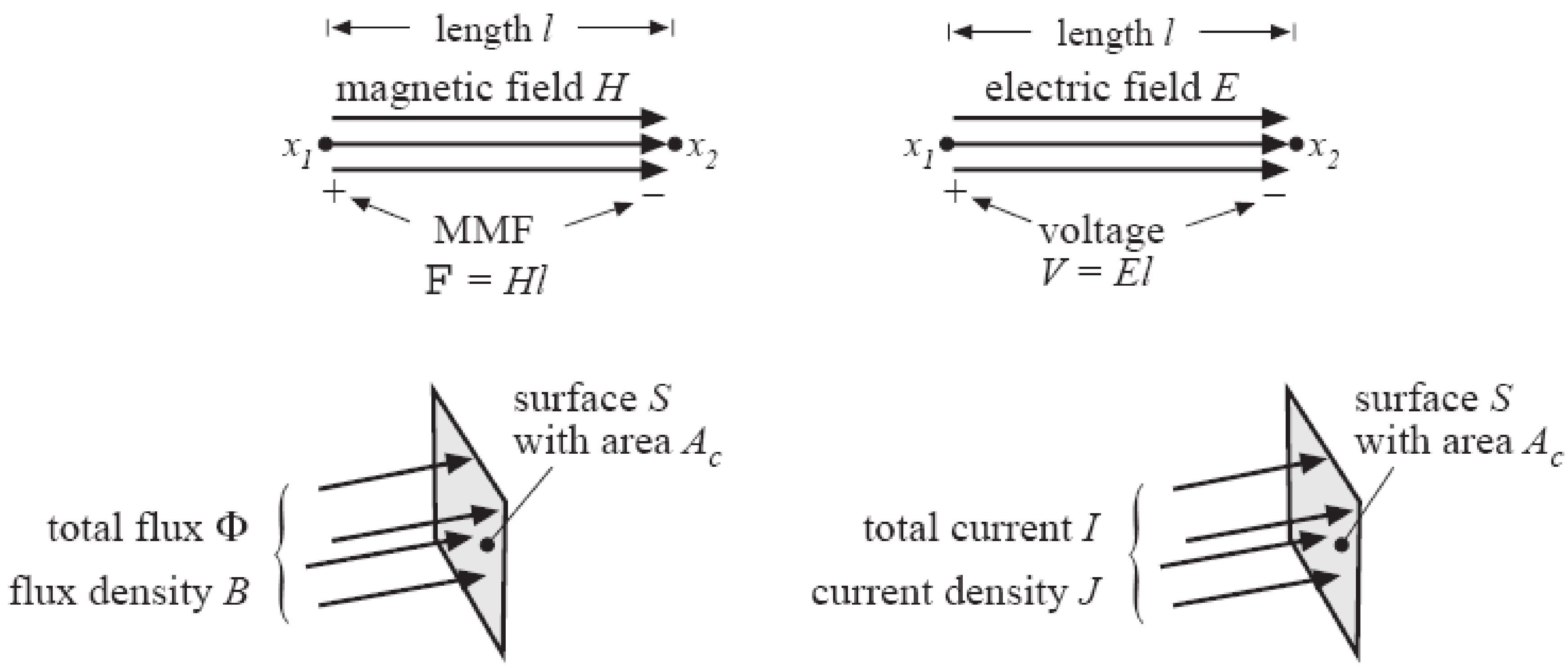
Highly nonlinear, with hysteresis and saturation

$\mu = \mu_r \mu_0$

Where μ_r typically is 1000-100000 in magnetic cores

Analogies

Magnetic quantities | *Electrical quantities*



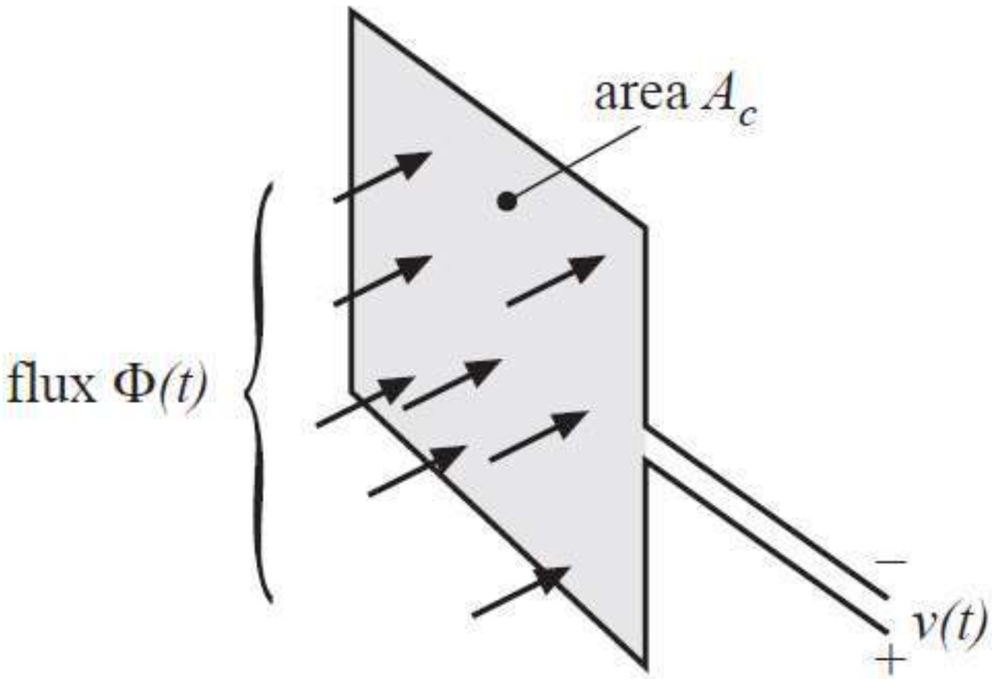
Faraday's law (again)

Voltage $v(t)$ is induced in a loop of wire by change in the total flux $\Phi(t)$ passing through the interior of the loop, according to

$$v(t) = \frac{d\Phi(t)}{dt}$$

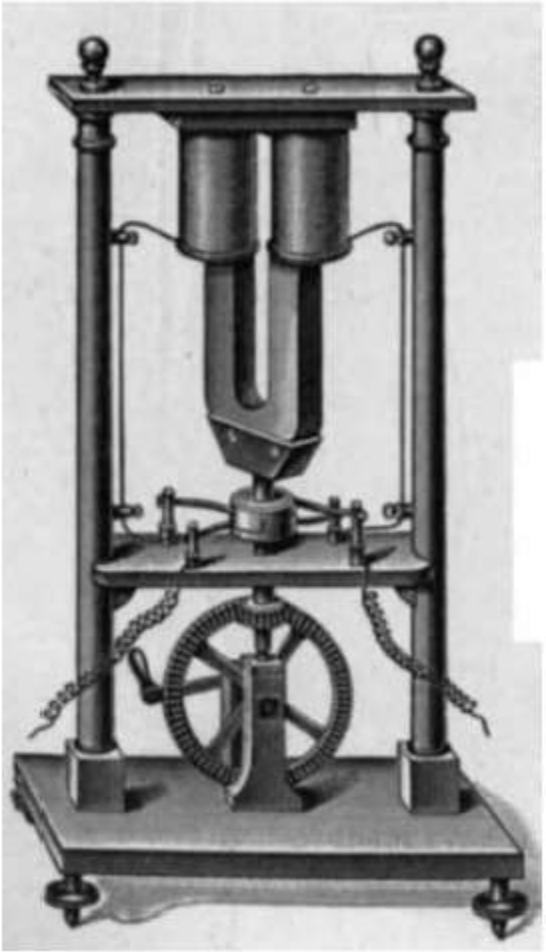
For uniform flux distribution, $\Phi(t) = B(t)A_c$ and hence

$$v(t) = A_c \frac{dB(t)}{dt}$$



Some of the first electrical machines

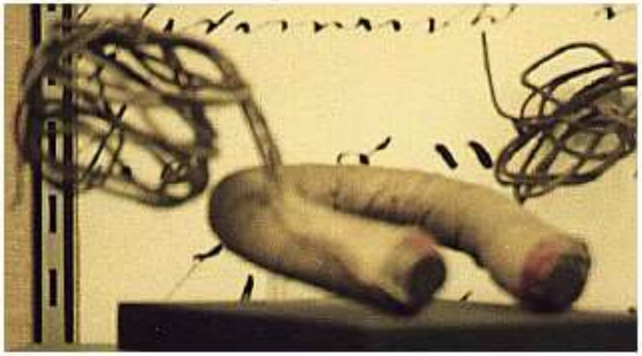
Alternator by Pixii 1832



Not useful because it makes AC

Improved to a DC machine
where Ampere proposed Pixii
to use a mechanical commutator

Note it is made with electromagnets
(Henry 1828)

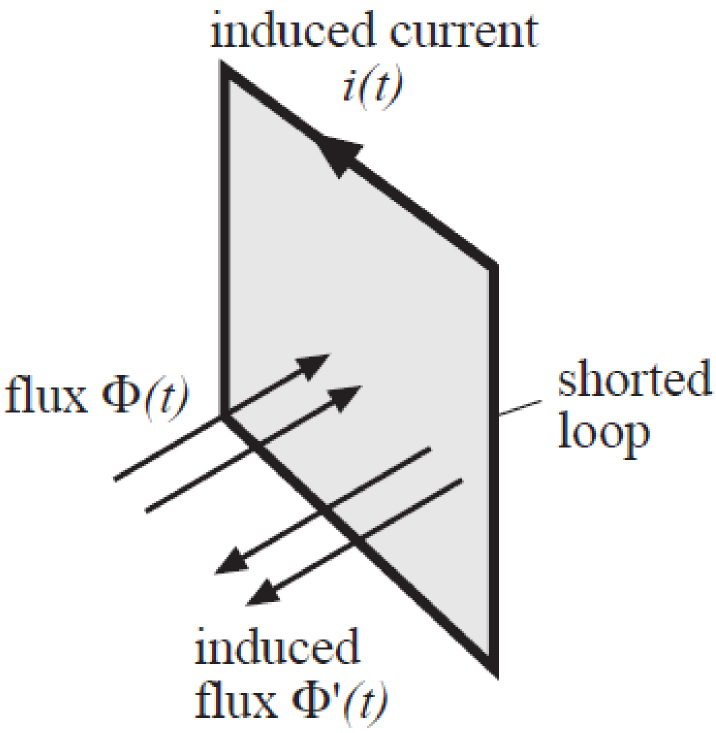


Letz's law (consequence of Maxwell's equation)

The voltage $v(t)$ induced by the changing flux $\Phi(t)$ is of the polarity that tends to drive a current through the loop to counteract the flux change.

Example: a shorted loop of wire

- Changing flux Φ(t) induces a voltage v(t) around the loop
- This voltage, divided by the impedance of the loop conductor, leads to current i(t)
- This current induces a flux Φ'(t), which tends to oppose changes in Φ(t)



Ampere's law (again)

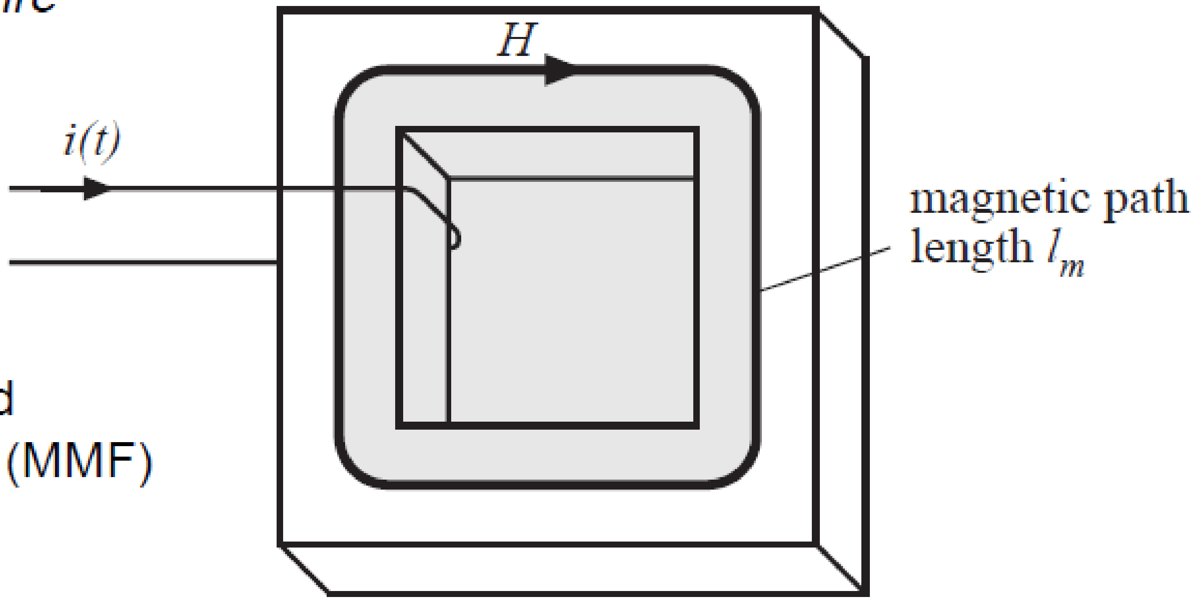
The net MMF around a closed path is equal to the total current passing through the interior of the path:

$$\oint_{closed\ path} \boldsymbol{H} \cdot \boldsymbol{dl} = total\ current\ passing\ through\ interior\ of\ path$$

Example: magnetic core. Wire carrying current $i(t)$ passes through core window.

- Illustrated path follows magnetic flux lines around interior of core
- For uniform magnetic field strength $H(t)$, the integral (MMF) is $H(t)l_m$. So

$$F(t) = H(t)\, l_m = i(t)$$

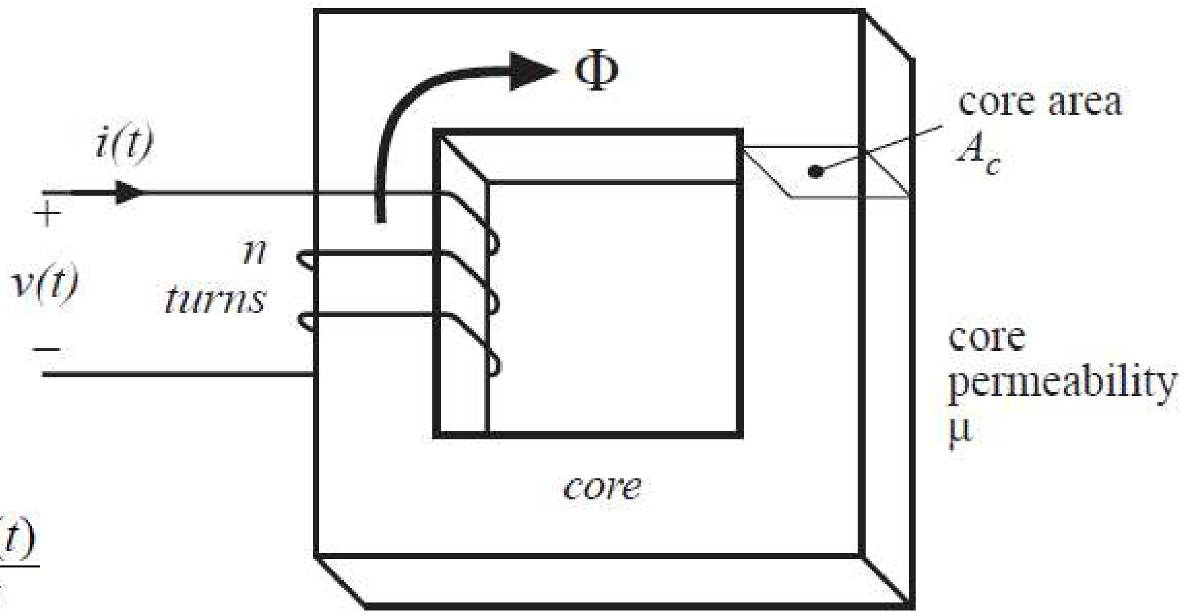


Example : Inductor

Faraday's law:

For each turn of wire, we can write

$$v_{turn}(t) = \frac{d\Phi(t)}{dt}$$



Total winding voltage is

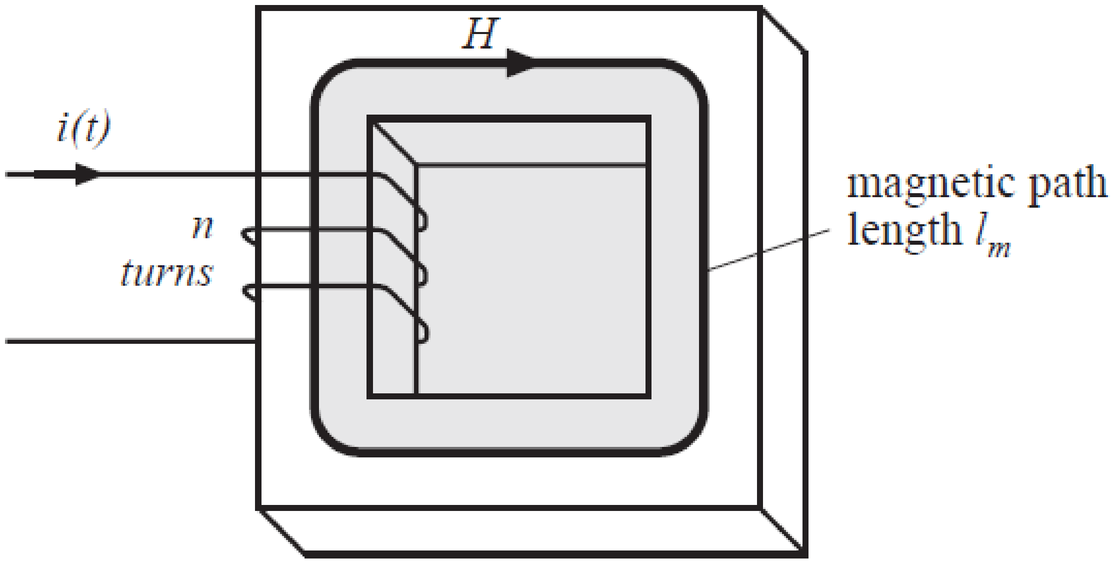
$$v(t) = n\, v_{turn}(t) = n\, \frac{d\Phi(t)}{dt}$$

Express in terms of the average flux density $B(t) = \Phi(t)/A_c$

$$v(t) = n\, A_c \frac{dB(t)}{dt}$$

Inductor example

Choose a closed path which follows the average magnetic field line around the interior of the core. Length of this path is called the *mean magnetic path length* l_m.



For uniform field strength $H(t)$, the core MMF around the path is $H\ l_m$.

Winding contains n turns of wire, each carrying current $i(t)$. The net current passing through the path interior (i.e., through the core window) is $ni(t)$.

From Ampere's law, we have

$$H(t)\ l_m = n\ i(t)$$

Inductor example

We have:

No saturation and hysteresis

$$v(t) = n\,A_c \frac{dB(t)}{dt} \qquad H(t)\,l_m = n\,i(t) \qquad B = \mu\,H$$

Eliminate B and H, and solve for relation between v and i.

$$v(t) = \mu\,n\,A_c \frac{dH(t)}{dt} \longrightarrow v(t) = \frac{\mu\,n^2 A_c}{l_m}\frac{di(t)}{dt}$$

which is of the form

$$v(t) = L\frac{di(t)}{dt}$$

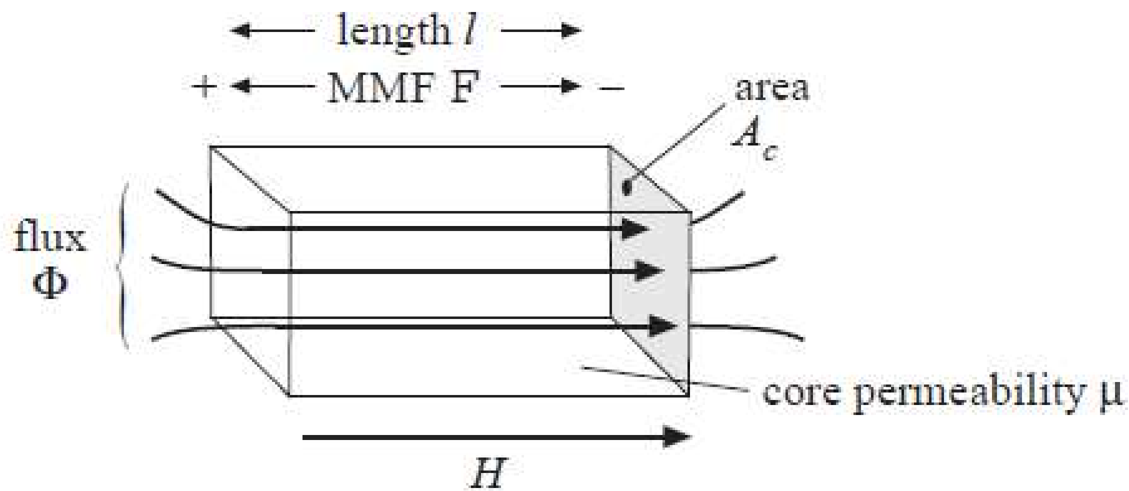
with

$$L = \frac{\mu\,n^2 A_c}{l_m}$$

—an inductor

Magnetic circuit model

Uniform flux and magnetic field inside a rectangular element:

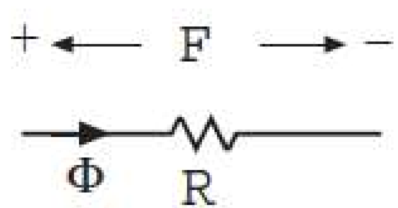


MMF between ends of element is $F = H\,l$

Since $H = B/\mu$ and $B = \Phi/Ac$, we can express F as

$$F = \frac{l}{\mu A_c}\Phi \qquad \text{with} \qquad R = \frac{l}{\mu A_c}$$

A corresponding model:



R = reluctance of element

Ohm's law in magnetic circuits

Magnetic circuit model

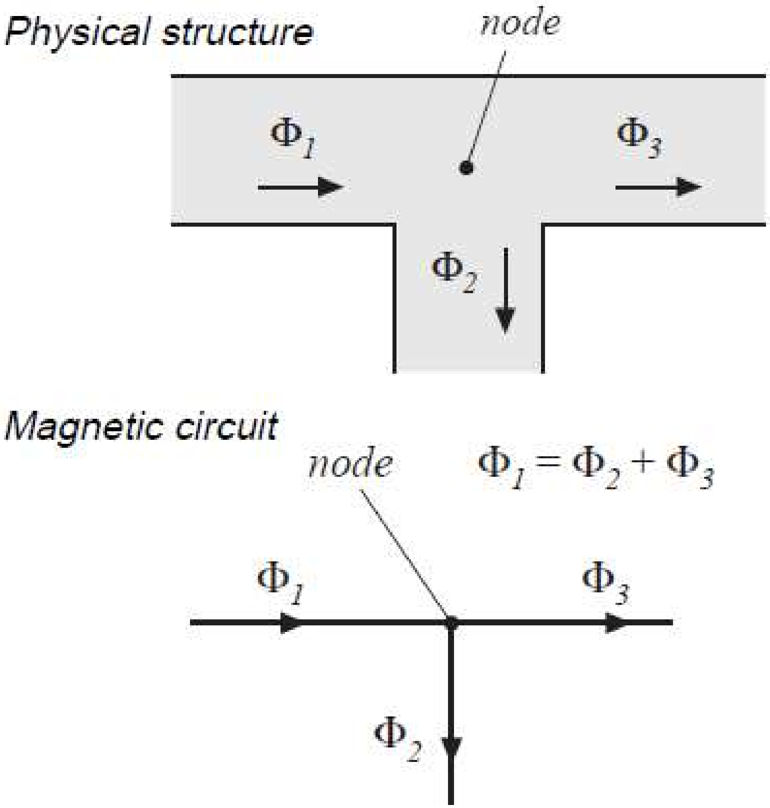
What about Kirchoff's current law (KCL)

Gauss law

Divergence of $\boldsymbol{B} = 0$

Flux lines are continuous and cannot end

Total flux entering a node must be zero



Magnetic circuit model

What about Kirchoff's voltage law (KVL)

Follows from Ampere's law:

$$\oint_{closed\ path} \boldsymbol{H} \cdot d\boldsymbol{l} = total\ current\ passing\ through\ interior\ of\ path$$

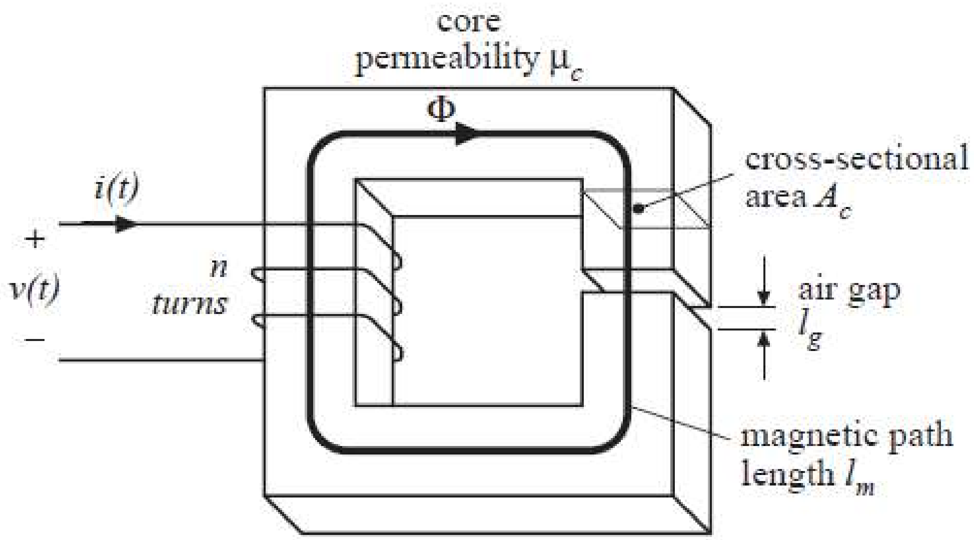
Left-hand side: sum of MMF's across the reluctances around the closed path

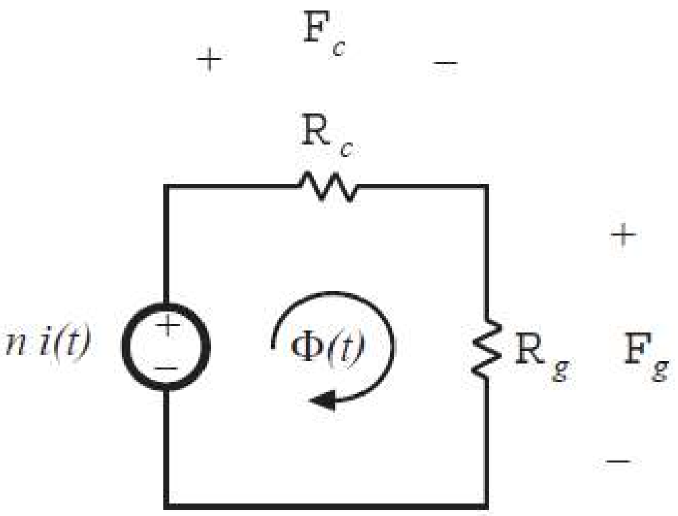
Right-hand side: currents in windings are sources of MMF's. An n-turn winding carrying current $i(t)$ is modeled as an MMF (voltage) source, of value $ni(t)$.

Total MMF's around the closed path add up to zero.

Ampere law is similar to Kirchoff's voltage law

Magnetic circuit model (Steel and air-gap)





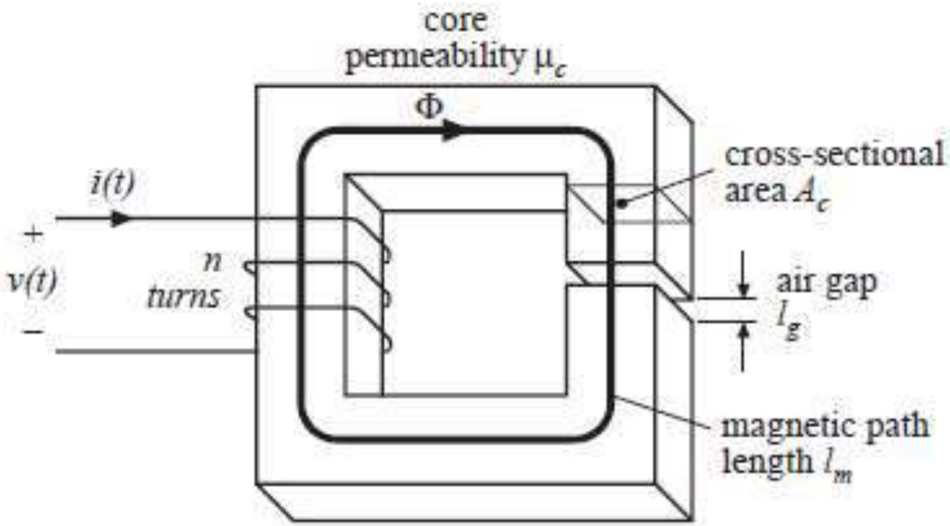
$$F_c + F_g = n\,i$$

$$n\,i = \Phi\left(R_c + R_g\right)$$

$$R_c = \frac{l_c}{\mu A_c}$$

$$R_g = \frac{l_g}{\mu_0 A_c}$$

Magnetic circuit model (Steel and air-gap)

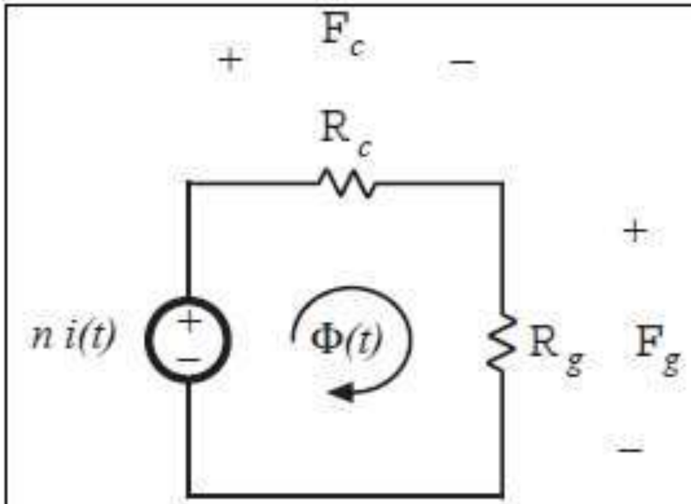


Faraday's law: $v(t) = n \dfrac{d\Phi(t)}{dt}$

Substitute for Φ: $v(t) = \dfrac{n^2}{R_c + R_g} \dfrac{di(t)}{dt}$

Hence inductance is

$$L = \frac{n^2}{R_c + R_g}$$



$$n\,i = \Phi\left(R_c + R_g\right)$$

$$R_c = \frac{l_c}{\mu A_c}$$

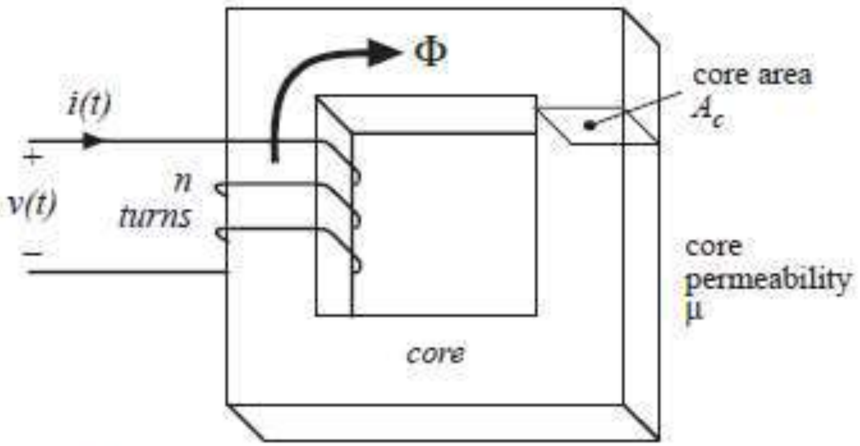
$$R_g = \frac{l_g}{\mu_0 A_c}$$

Core losses

Hysteresis losses

Energy per cycle W flowing into n-turn winding of an inductor, excited by periodic waveforms of frequency f:

$$W = \int_{\text{one cycle}} v(t)i(t)dt$$



Relate winding voltage and current to core B and H via Faraday's law and Ampere's law:

$$v(t) = n A_c \frac{dB(t)}{dt} \qquad H(t)\, l_m = n\, i(t)$$

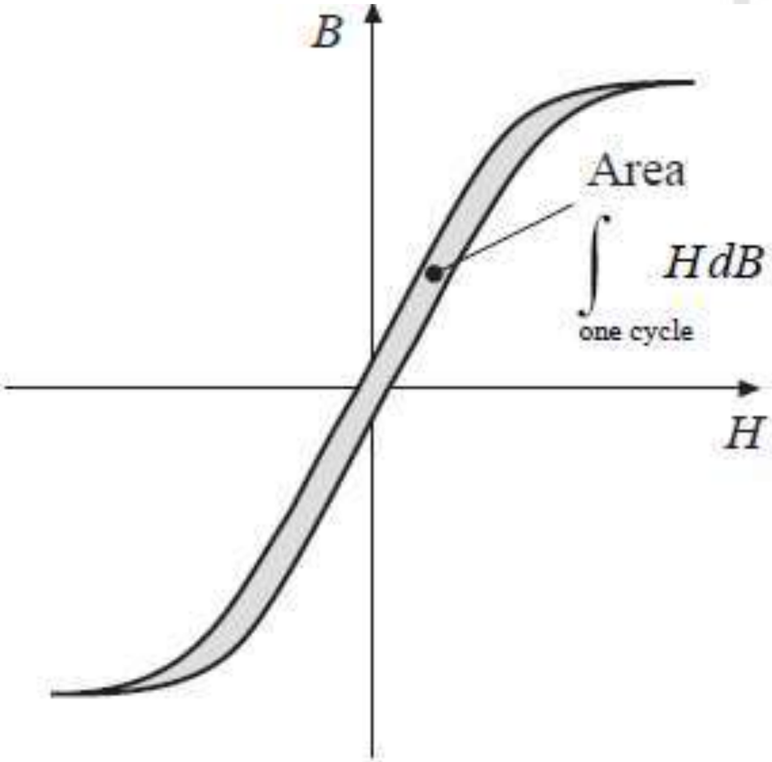
Substitute into integral:

$$W = \int_{\text{one cycle}} \left(nA_c \frac{dB(t)}{dt}\right)\left(\frac{H(t)l_m}{n}\right)dt$$

$$= \left(A_c l_m\right) \int_{\text{one cycle}} H\, dB$$

Hysteresis losses

$$W = (A_c l_m) \int_{\text{one cycle}} H\,dB$$



The term $A_c l_m$ is the volume of the core, while the integral is the area of the B-H loop.

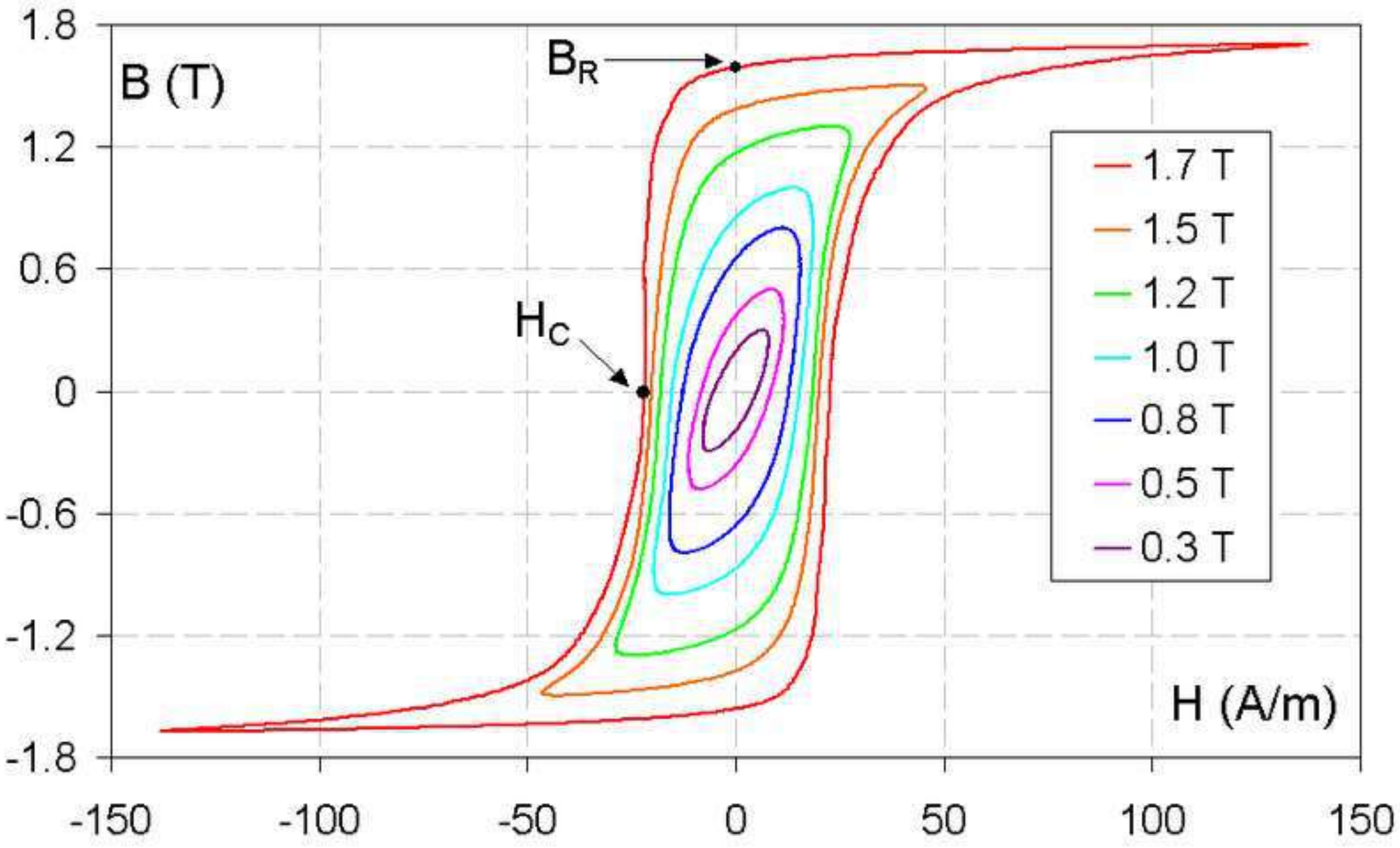
(energy lost per cycle) = (core volume) (area of B-H loop)

$$P_H = (f)(A_c l_m) \int_{\text{one cycle}} H\,dB$$

Hysteresis loss is directly proportional to applied frequency

Hysteresis losses

Hysteresis curve at various H-fields



Hysteresis losses

- Hysteresis loss varies directly with applied frequency
- Dependence on maximum flux density: how does area of *B*-*H* loop depend on maximum flux density (and on applied waveforms)? Emperical equation (Steinmetz equation):

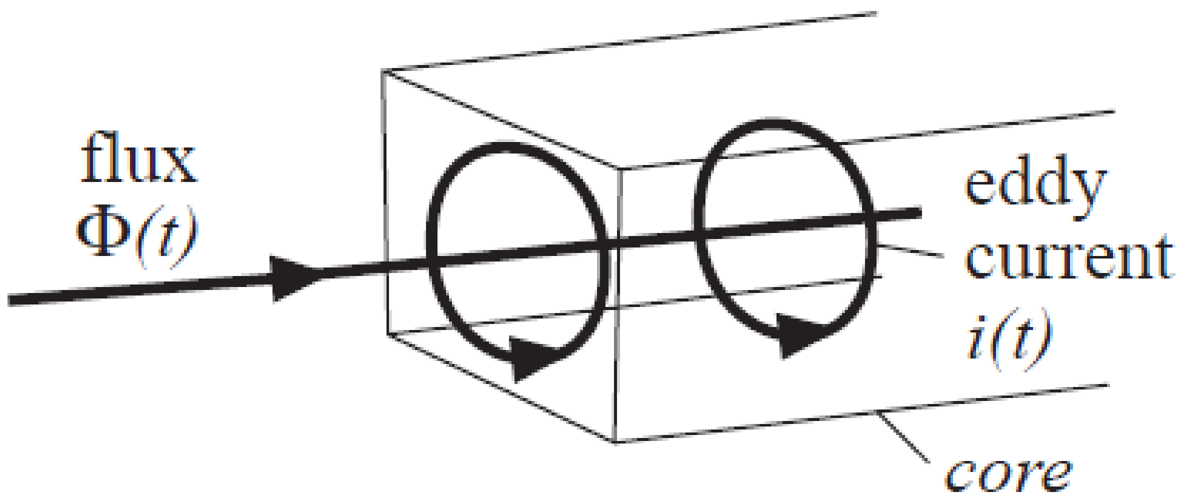
$$P_H = K_H f B_{\max}^{\alpha} (core\ volume)$$

The parameters K_H and α are determined experimentally.

Dependence of P_H on B_{max} is predicted by the theory of magnetic domains.

Eddy current losses

Magnetic core materials are reasonably good conductors of electric current. Hence, according to Lenz's law, magnetic fields within the core induce currents ("eddy currents") to flow within the core. The eddy currents flow such that they tend to generate a flux which opposes changes in the core flux $\Phi(t)$. The eddy currents tend to prevent flux from penetrating the core.

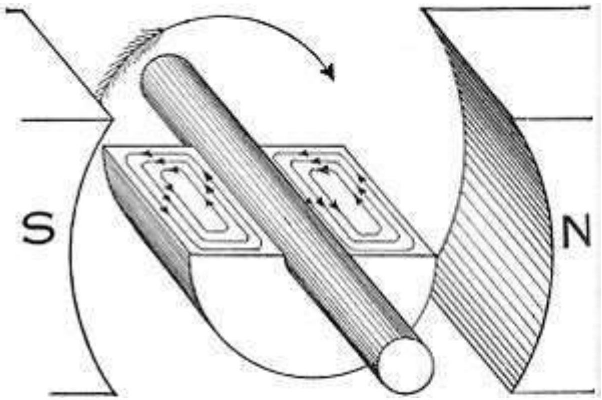


Eddy current loss $i^2(t)R$

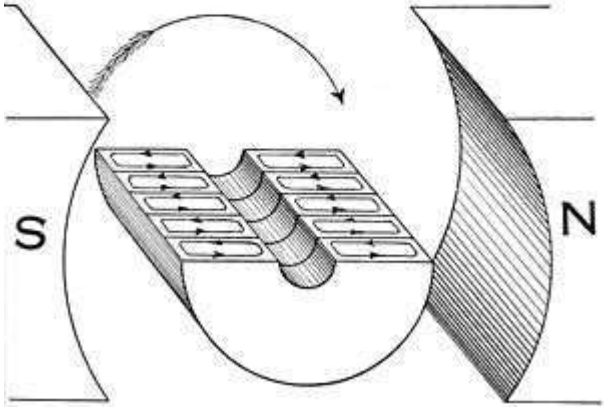
Eddy current losses

How do we reduce Eddy current losses

SOLID



LAMINATED

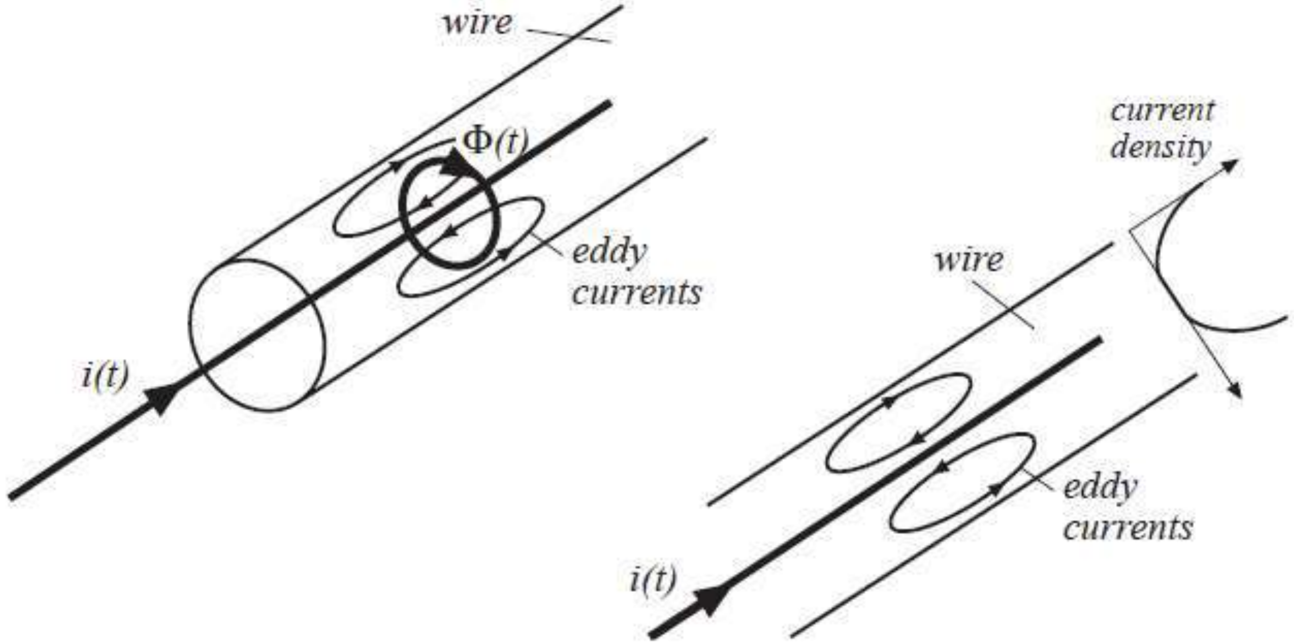


Eddy current losses

- Ac flux $\Phi(t)$ induces voltage $v(t)$ in core, according to Faraday's law. Induced voltage is proportional to derivative of $\Phi(t)$. In consequence, magnitude of induced voltage is directly proportional to excitation frequency f.
- If core material impedance Z is purely resistive and independent of frequency, $Z = R$, then eddy current magnitude is proportional to voltage: $i(t) = v(t)/R$. Hence magnitude of $i(t)$ is directly proportional to excitation frequency f.
- Eddy current power loss $i^2(t)R$ then varies with square of excitation frequency f.
- Classical Steinmetz equation for eddy current loss:

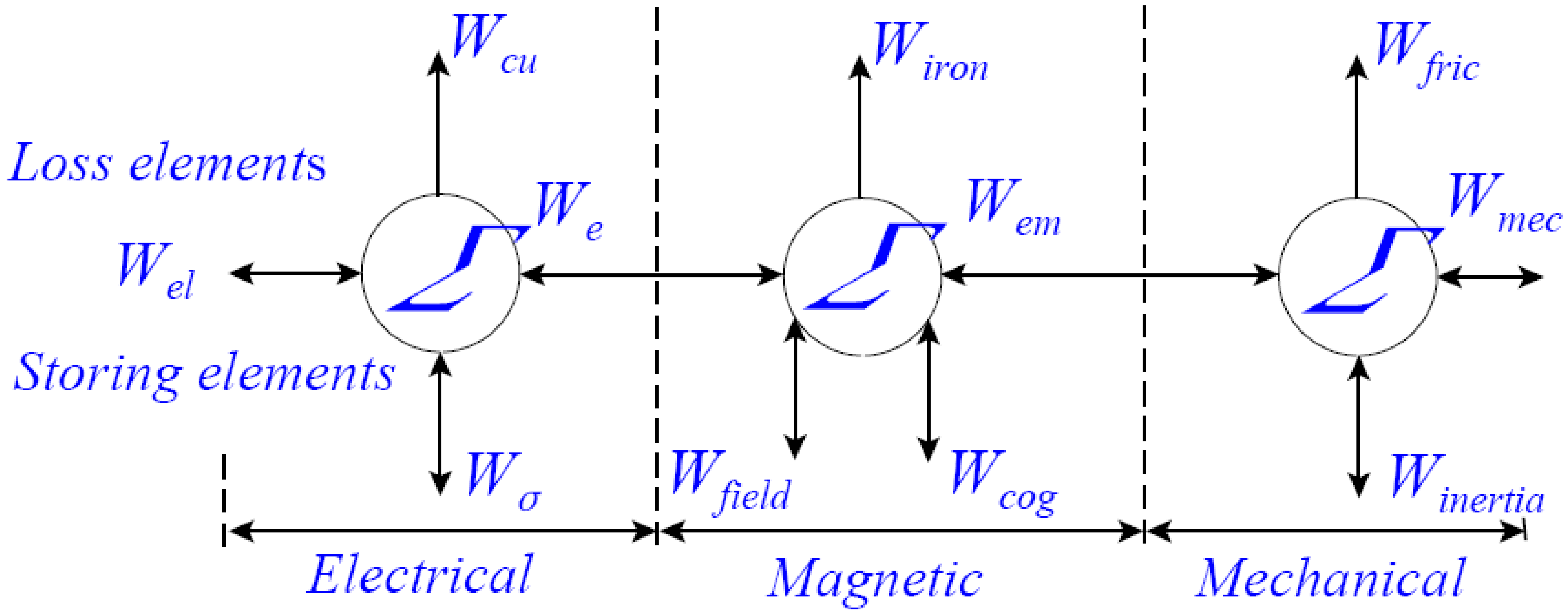
$$P_E = K_E f^2 B_{max}^2 (core\ volume)$$

Eddy current losses in windings



Can be a problem with thick wires
- Low voltage machines
- High speed machines

Universal modeling of terminal characteristic of electro-magnetic devices based on energy balance



Force, torque and power

1. Energy balance

$$W_e = W_{el} - W_{cu} - W_\sigma$$

$$W_{em} = W_e - W_{field} - W_{cog}$$

$$W_{load} = W_{em} - W_{fric} - W_{inertia}$$

$$\mathbf{W_{iron} = 0}$$

2. Voltage equation

$$u_j = R_j i_j + L_{\sigma j}\frac{di_j}{dt} + \frac{d\psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{dt}$$

3. Flux linkage

$$\psi_j(\theta_r, i_1, i_2, \ldots, i_n) = \psi_{j,e}(\theta_r, i_1, i_2, \ldots, i_n) + \psi_{j,pm}(\theta_r)$$

Approximation that $\psi_{j,pm}$ only depends on the position.
Temperature effects : Ex. NdFeB
B_r = -0.11 %/°C ; H_c = -0.60 %/°C

4. Applied energy to the field

$$W_e = W_{el} - W_{cu} - W_\sigma = \int \sum_{j=1}^{n} \left(u_j i_j - R_j i_j^2 - L_{\sigma,j} di\right) dt = \int \sum_{j=1}^{n} i_j d\psi_j(\theta_r, i_1, i_2, \ldots, i_n)$$

Force, torque and power

5. Field energy (based on 1 og 4)

$$W_{field} = W_e - W_{em} - W_{cog}$$
$$= \int \sum_{j=1}^{n} i_j d\psi_j(\theta_r, i_1, i_2, \ldots, i_n) - W_{em} - W_{cog}$$

6. Cogging energy on differential form

$$dW_{cog} = \tau_{cog} d\theta_r$$

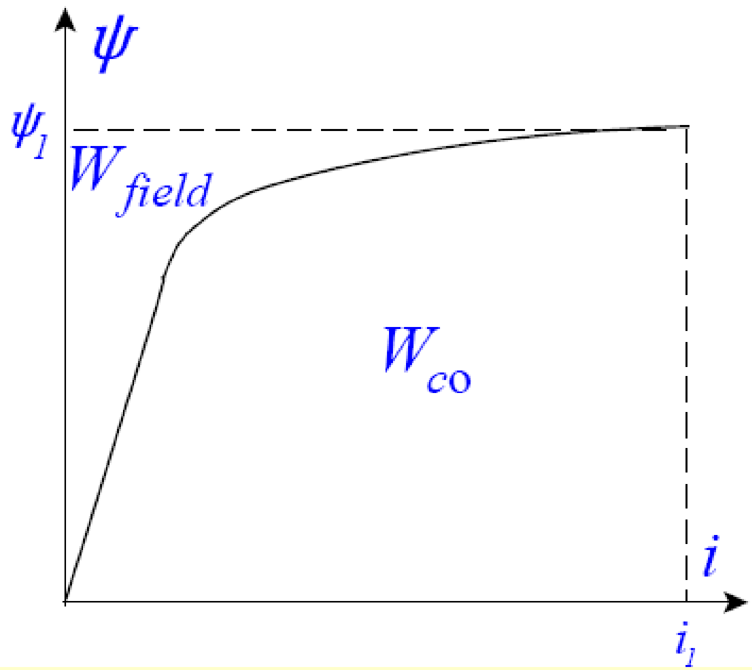
7. Field energy on differential form

$$dW_{field} = \sum_{j=1}^{n} i_j d\psi_j(\theta_r, i_1, i_2, \ldots, i_n) - \tau_{em} d\theta_r - \tau_{cog} d\theta_r$$

8. Field energy by fixing the position ($d\theta_r = 0$)

$$W_{field} = \int \sum_{j=1}^{n} i_j d\psi_j(\theta_r, i_1, i_2, \ldots, i_n)$$

Graphical presentation of the field energy for a single excitated circuit without permanent magnets.



Force, torque and power

9. Mathematical description of the CO-energy

$$W_{co} = \sum_{j=1}^{n} i_j \psi_j(\theta_r, i_1, i_2, \ldots, i_n) - W_{field} = \int \sum_{j=1}^{n} \psi_j(\theta_r, i_1, i_2, \ldots, i_n) di_j$$

10. Re-written equation 7 differential form with help of partial differentiation

$$dW_{field} = \sum_{j=1}^{n} \frac{\partial W_{field}}{\partial i_j} di_j + \frac{\partial W_{field}}{\partial \theta_r} d\theta_r$$

$$d\psi_j(\theta_r, i_1, i_2, \ldots, i_n) = \sum_{k=1}^{n} \frac{\partial \psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{\partial i_k} di_k + \frac{\partial \psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{\partial \theta_r} d\theta_r$$

Force, torque and power

11. Insertion of equation 10 in equation 7

$$\tau_{em} d\theta_r = \sum_{j=1}^{n} i_j \left[\sum_{k=1}^{n} \frac{\partial \psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{\partial i_k} di_k + \frac{\partial \psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{\partial \theta_r} d\theta_r \right]$$

$$- \sum_{j=1}^{n} \frac{\partial W_{field}}{\partial i_j} di_j - \frac{\partial W_{field}}{\partial \theta_r} d\theta_r - \tau_{cog} d\theta_r$$

$$= \left[\sum_{j=1}^{n} i_j \frac{\partial \psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{\partial \theta_r} - \frac{\partial W_{field}}{\partial \theta_r} - \tau_{cog} \right] d\theta_r$$

$$+ \sum_{j=1}^{n} \left[\sum_{k=1}^{n} i_j \frac{\partial \psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{\partial i_k} di_k - \frac{\partial W_{field}}{\partial i_j} di_j \right]$$

12. Equalizing coefficients (equation 11 and equation 7)

$$\tau_{em} = \sum_{j=1}^{n} i_j \frac{\partial \psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{\partial \theta_r} - \frac{\partial W_{field}}{\partial \theta_r} - \tau_{cog}$$

Force, torque and power

13. With help of equation 9

$$\frac{\partial W_c}{\partial \theta_r} = \sum_{j=1}^{n} i_j \frac{\partial \psi_j(\theta_r, i_1, i_2, \ldots, i_n)}{\partial \theta_r} - \frac{\partial W_{field}}{\partial \theta_r}$$

14. Insertion of equation 13 in equation 12 a final torque equation is derived :

$$\tau_{em} = \frac{\partial W_c}{\partial \theta_r} - \tau_{cog}$$

Example with a single excitated circuit

$$\tau_{em} = \frac{\partial \int_0^{i_1} \psi_1(\theta_r, \xi) d\xi}{\partial \theta_r} - \tau_{cog}$$

$$= \frac{\partial \int_0^{i_1} \psi_{1,e}(\theta_r, \xi) d\xi}{\partial \theta_r} + \frac{\partial \int_0^{i_1} \psi_{1,pm}(\theta_r) d\xi}{\partial \theta_r} - \tau_{cog}$$

$$= \frac{\partial \int_0^{i_1} \psi_{1,e}(\theta_r, \xi) d\xi}{\partial \theta_r} + i_1 \frac{d \psi_{1,pm}(\theta_r)}{d\theta_r} - \tau_{cog}$$

Force, torque and power

If it is assumed that there is no saturation

$$\psi_{1,e}(\theta_r, i_1) = L_{11,e}(\theta_r) \cdot i_1$$

Torque equation

$$\tau_{em} = \frac{1}{2}\frac{\partial L_{11}(\theta_r)}{\partial \theta_r} i_1^2 + i_1 \frac{d\psi_{1,pm}(\theta_r)}{d\theta_r} - \tau_{cog}$$

Reluctance torque
Winding ⇨ steel

Permanent magnet torque
Winding ⇨ Permanent magnet

Cogging torque
Permanent magnet ⇨ steel

Missing torque types ?

Steel ⇨ Steel : No torque, nothing is active
winding ⇨ winding : Excitation torque (Ex DC motor), requires 2 current circuits
PM ⇨ PM : I have only seen a single patent with such a torque type

Voltage equation

$$u_1 = R_1 i_1 + L_{\sigma,1}\frac{d i_1}{d t} + \frac{d(L_{11}(\theta_r) i_1)}{d t} + \frac{d\psi_{1,pm}(\theta_r)}{d t}$$

$$= R_1 i_1 + L_{\sigma,1}\frac{d i_1}{d t} + i_1 \frac{d L_{11}(\theta_r)}{d t} + L_{11}(\theta_r)\frac{d i_1}{d t} + \frac{d\psi_{1,pm}(\theta_r)}{d t}$$

Example with 3 excitations

Torque equation

$$\tau_{em} = \frac{\partial \int \sum_{j=1}^{3} \psi_j(\theta_r, i_1, i_2, i_3) di_j}{\partial \theta_r} - \tau_{cog}$$

$$= \frac{\partial \int \sum_{j=1}^{3} \psi_{j,e}(\theta_r, i_1, i_2, i_3) di_j}{\partial \theta_r} + \frac{\partial \int \sum_{j=1}^{3} \psi_{j,pm}(\theta_r) di_j}{\partial \theta_r} - \tau_{cog}$$

$$= \frac{\partial \int \psi_{1,e}(\theta_r, i_1, i_2, i_3) di_1 + \psi_{2,e}(\theta_r, i_1, i_2, i_3) di_2 + \psi_{3,e}(\theta_r, i_1, i_2, i_3) di_3}{\partial \theta_r}$$

$$+ \frac{\partial \int \psi_{1,pm}(\theta_r) di_1 + \psi_{2,pm}(\theta_r) di_2 + \psi_{3,pm}(\theta_r) di_3}{\partial \theta_r} - \tau_{cog}$$

Force, torque and power

The magnetic field energy in the circuit do not depend on how the flux linkage is obtained. The integration can thus be done in 3 steps :

$$Step\ 1\ (di_1)\ :\ i_1 = i_1\ ,\ i_2 = 0\ ,\ i_3 = 0\ ,\ di_2 = 0\ ,\ di_3 = 0$$
$$Step\ 2\ (di_2)\ :\ i_1 = i_1\ ,\ i_2 = i_2\ ,\ i_3 = 0\ ,\ di_1 = 0\ ,\ di_3 = 0$$
$$Step\ 3\ (di_3)\ :\ i_1 = i_1\ ,\ i_2 = i_2\ ,\ i_3 = i_3\ ,\ di_1 = 0\ ,\ di_2 = 0$$

The torque is hereby reduced to :

$$\tau_{em} = \frac{\partial \int_0^{i_1} \psi_{1,e}(\theta_r, \xi, 0, 0)\, d\xi}{\partial \theta_r} + \frac{\partial \int_0^{i_2} \psi_{2,e}(\theta_r, i_1, \xi, 0)\, d\xi}{\partial \theta_r} + \frac{\partial \int_0^{i_3} \psi_{3,e}(\theta_r, i_1, i_2, \xi)\, d\xi}{\partial \theta_r}$$
$$+ \frac{d\psi_{1,pm}(\theta_r)}{d\theta_r} i_1 + \frac{d\psi_{2,pm}(\theta_r)}{d\theta_r} i_2 + \frac{d\psi_{3,pm}(\theta_r)}{d\theta_r} i_3 - \tau_{cog}$$

EXTREME COMPLICATED. Model parameters depend on four parameters.

The conclusion is that people in many cases avoids the saturated couplings such both design and control is simplified.

Force, torque and power

Unsaturated machine

$$\psi_{1,e}(\theta_r, i_1, i_2, i_3) = L_{11,e}(\theta_r)\, i_1 + L_{12,e}(\theta_r)\, i_2 + L_{13,e}(\theta_r)\, i_3$$
$$\psi_{2,e}(\theta_r, i_1, i_2, i_3) = L_{21,e}(\theta_r)\, i_1 + L_{22,e}(\theta_r)\, i_2 + L_{23,e}(\theta_r)\, i_3$$
$$\psi_{3,e}(\theta_r, i_1, i_2, i_3) = L_{31,e}(\theta_r)\, i_1 + L_{32,e}(\theta_r)\, i_2 + L_{33,e}(\theta_r)\, i_3$$

Torque equation

$$\tau_{em} = \frac{1}{2}\frac{dL_{11}(\theta_r)}{d\theta_r} i_1^2 + \frac{1}{2}\frac{dL_{22}(\theta_r)}{d\theta_r} i_2^2 + \frac{1}{2}\frac{dL_{33}(\theta_r)}{d\theta_r} i_3^2 +$$
$$+ \frac{dL_{12}(\theta_r)}{d\theta_r} i_1 i_2 + \frac{dL_{13}(\theta_r)}{d\theta_r} i_1 i_3 + \frac{dL_{23}(\theta_r)}{d\theta_r} i_2 i_3 +$$

Winding ⇒ winding torque

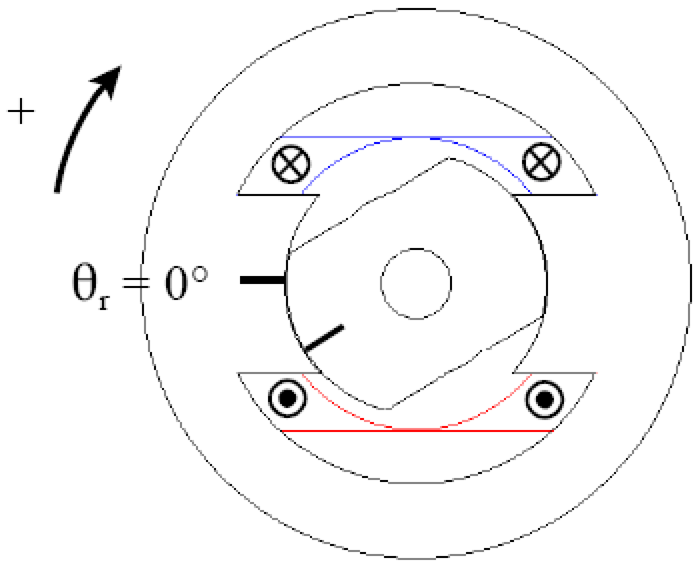
$$+ \frac{d\psi_{1,pm}(\theta_r)}{d\theta_r} i_1 + \frac{d\psi_{2,pm}(\theta_r)}{d\theta_r} i_2 + \frac{d\psi_{3,pm}(\theta_r)}{d\theta_r} i_3 - \tau_{cog}$$

Voltage equation (phase 1)

$$u_1 = R_1 i_1 + L_{\sigma,1}\frac{di_1}{dt} + i_1 \frac{dL_{11}(\theta_r)}{dt} + L_{11}(\theta_r)\frac{di_1}{dt} + i_2 \frac{dL_{12}(\theta_r)}{dt} + L_{12}(\theta_r)\frac{di_2}{dt}$$
$$+ i_3 \frac{dL_{13}(\theta_r)}{dt} + L_{13}(\theta_r)\frac{di_3}{dt} + \frac{d\psi_{1,pm}(\theta_r)}{dt}$$

Simplified machines

Single phase Switched Reluctance Motor



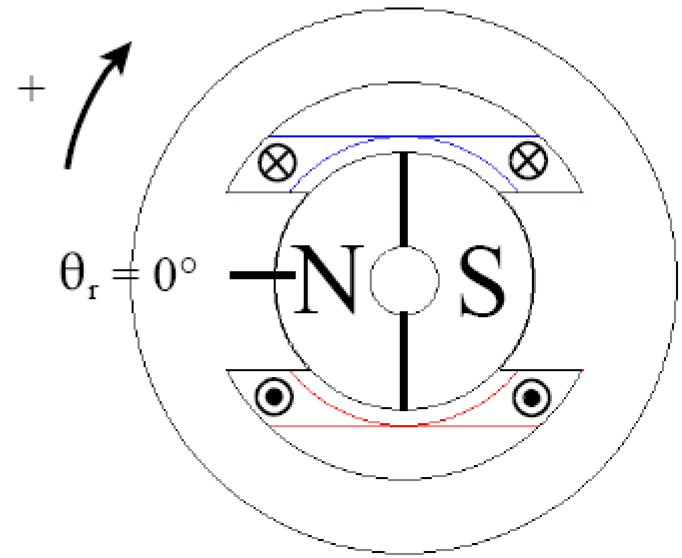
Voltage equation

$$u_1 = R_1 i_1 + i_1 \frac{dL_{11}(\theta_r)}{dt} + L_{11}(\theta_r)\frac{di_1}{dt}$$

Torque equation

$$\tau_{em} = \frac{1}{2}\frac{\partial L_{11}(\theta_r)}{\partial \theta_r} i_1^2$$

Single phase Brushless DC Motor



Voltage equation

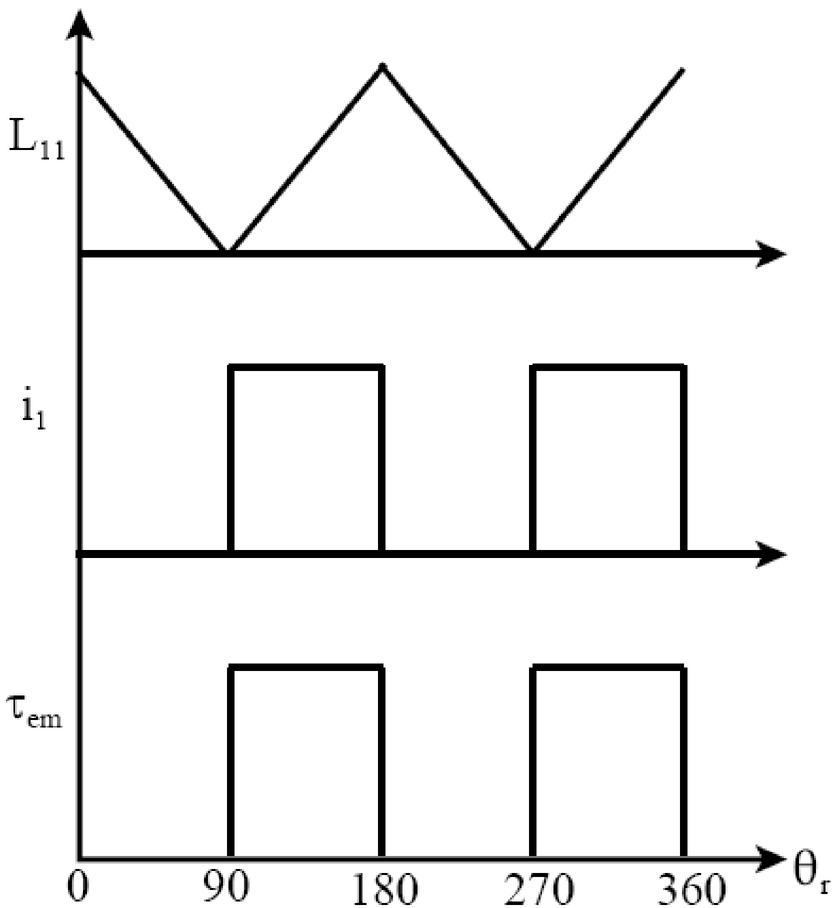
$$u_1 = R_1 i_1 + L_{11}\frac{di_1}{dt} + \frac{d\psi_{1,pm}(\theta_r)}{dt}$$

Torque equation

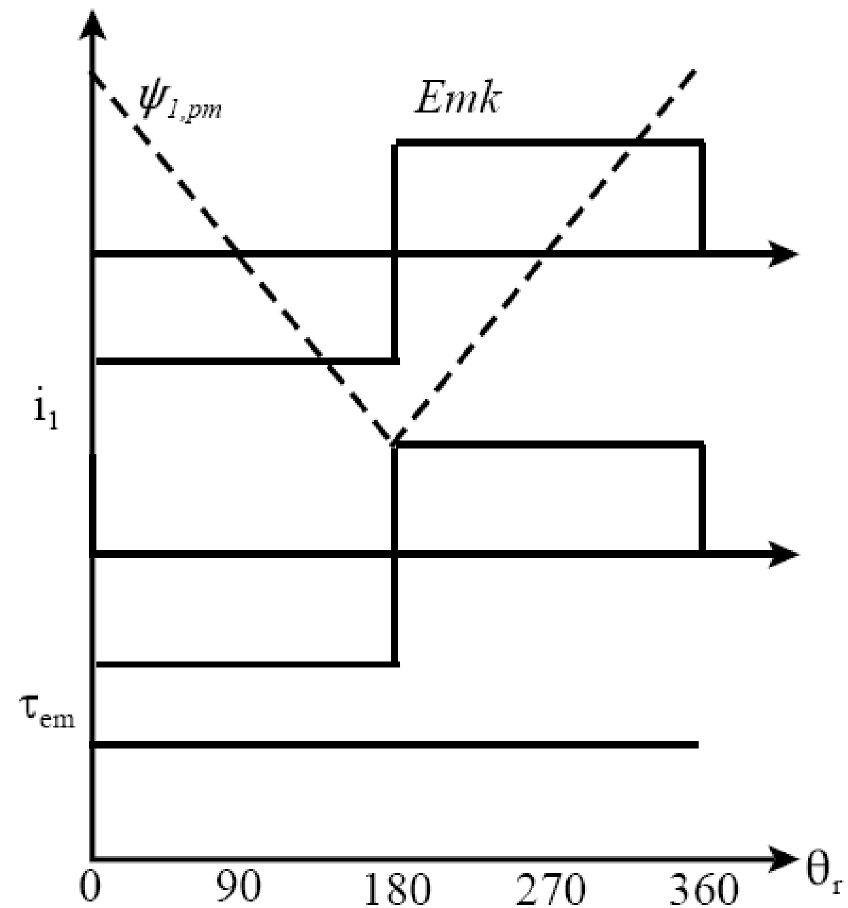
$$\tau_{em} = i_1 \frac{d\psi_{1,pm}(\theta_r)}{d\theta_r} - \tau_{cog}$$

Ideal characteristics

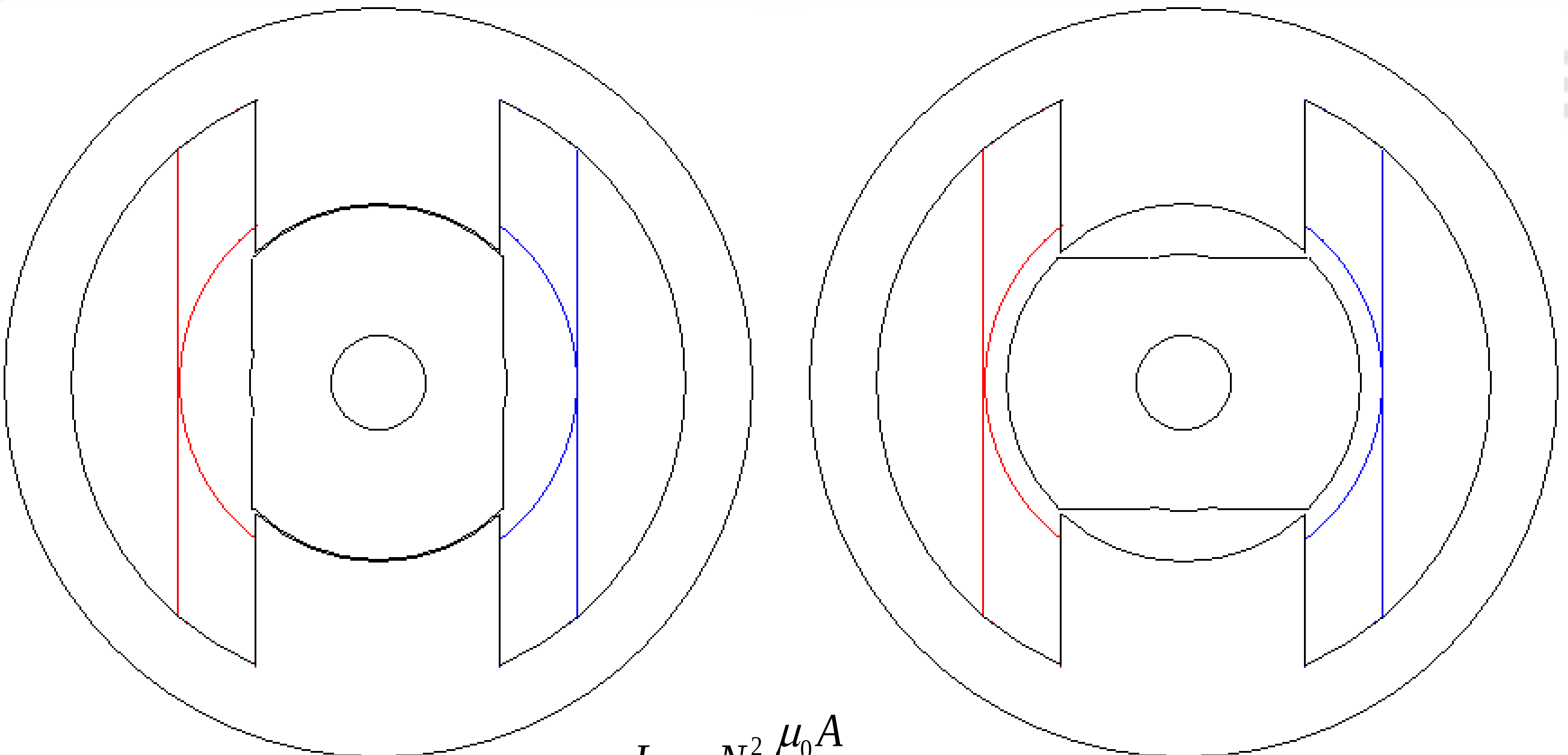
Single phase Switched Reluctance Motor



Single phase Brushless DC Motor



Electromagnetic gearing



$$E_L = \frac{1}{2} L I^2 \{ \ Inductor \ \ Energy \ \}$$

$$E_C = \frac{1}{2} C U^2 \{ \ Capacitor \ \ Energy \ \}$$

$$E_J = \frac{1}{2} J \omega^2 \{ \ Rotational \ \ Energy \ \}$$

$$E_m = \frac{1}{2} M v^2 \{ \ translational \ \ Energy \ \}$$

$$L_A = N^2 \frac{\mu_0 A}{l_g}$$

L_{gu} larger than $L_g \approx \infty$

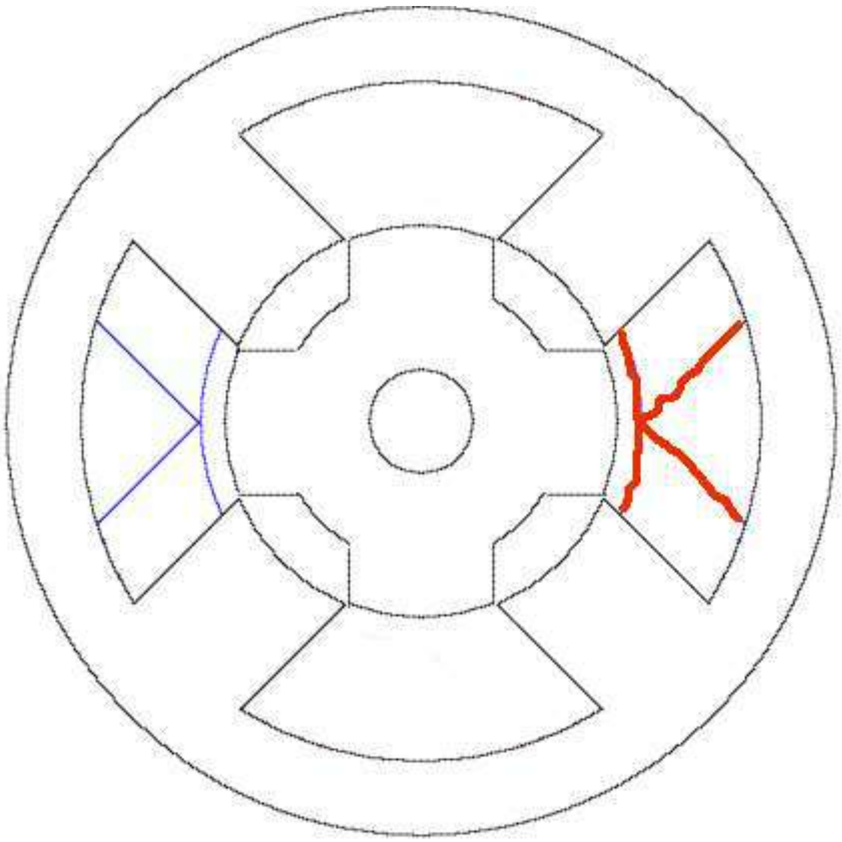
$$L_U = N^2 \frac{\mu_0 A_u}{l_{gu}}$$

$$\Delta E = \frac{1}{2} L_A I^2 - \frac{1}{2} L_U I^2 \approx \frac{1}{2} L_A I^2$$

$$\tau = \frac{\Delta E}{\Delta \theta} = \frac{1}{2} \frac{L_A}{\Delta \theta} I^2$$

$\Delta\theta = 90$

Electromagnetic gearing

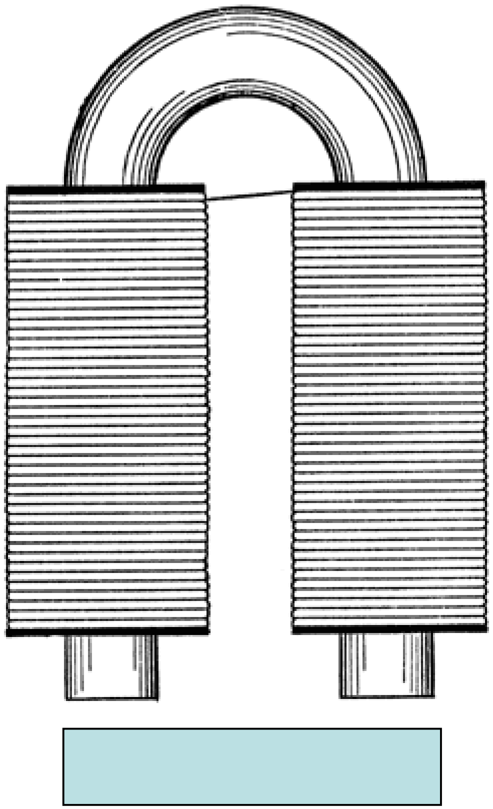


Same winding but now enclosing two poles : L_A is the same
(if end winding is neglected the Resistance is also the same)

$\Delta\theta=45$

Torque doubling

Normal forces



Ex.
Changing the air gap from 0.3 mm to 0.15 mm
Gives a doubling of inductance
The Δx is very small i.e a large Force

In electrical machines the normal forces versus the tangential force (torque) is typically a factor 10.